Third Small Workshop on Operator Theory

June 28 – July 1, 2008
Krakow, Poland

The Workshop is organized by:
University of Agriculture in Krakow
University of Lille 1
Institute of Mathematics of the Polish Academy of Sciences

The Scientific Committee:

Ernst Albrecht (Saarbruecken, Germany)
Hari Bercovici (Bloomington, USA)
Petru A. Cojuhari (Krakow, Poland)
Gustavo Corach (Buenos Aires, Argentina)
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Martin Mathieu (Belfast, Northern Ireland)
Mostafa Mbekhta (Lille, France)
Vladimír Müller (Prague, Czech Republic)
Marek Ptak (Krakow, Poland)
Jan Stochel (Krakow, Poland)
Franciszek H. Szafraniec (Krakow, Poland)
Jaroslav Zemánek (Warsaw, Poland)

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On Saturday (June 28) the conference will take place in the Conference Building of University of Agriculture in Krakow (ul. Balicka 253). The transportation from the hotel "U Pana Cogito" is fixed at 8:30. A bus will wait next to the entrance. The transportation from the student hotel "Nawojka" is fixed at 8:40.

From Monday (June 30) the conference will take place in the Main Building of University of Agriculture in Krakow (al. Mickiewicza 24/28).

On Sunday (June 29) we plan an excursion to Pieniny Mountains. The bus will leave at 7:50 from the parking of Department of Mathematics (ul. Ingardena) and at 8:00 from the hotel "U Pana Cogito". If the weather is fine we will take raft down the Dunajec river (cost around 40 PLN) and then after lunch we will go to Homole Gully (tickets around 2 PLN). In case of bad weather we plan to visit 4 castles in: Dobczyce, Nowy Wiśnicz, Czorsztyn and Nidzica (tickets around 10 PLN each). If you are interested, please sign a list which will be passed on Saturday, June 28.

On Monday a conference dinner will be held at the "Zajazd Zazamcze" in Ojców. The bus is arranged at 16:30 on the parking of Department of Mathematics (ul. Ingardena).

Registration office will be open on Saturday from 8:30 till 9:20 at the Conference Building of University of Agriculture in Krakow (ul. Balicka 253).

Internet access is available on Monday and Tuesday on the fourth and the fifth floor of the Main Building of University of Agriculture in Krakow (near Room 434 and Room 532).

Programme of the conference

Saturday, June 28th
Conference Hall: ul. Balicka 253

8:30 - 9:20	Registration	
9:30 - 9:40	Opening	
	Plenary session Chair: Marek Ptak	
9:40 - 10:10	**Ernst Albrecht** *Chebyshev polynomials for operators*	
10:15 - 10:55	**Hari Bercovici** *The extreme points of the Horn body*	
10:55 - 11:20	**Coffee break**	
	Chair: Vladimír Müller	
11:20 - 12:00	**Aurelian Gheondea** *Absolute continuity of operator valued completely positive maps*	
12:05 - 12:45	**Petru A. Cojuhari** *Spectral analysis of an operator pencil with applications to linear transport*	
12:50 - 13:10	**Cristina Câmara** *Almost periodic factorization for a class of triangular matrix symbols*	
13:10 - 15:00	**Lunch**	
15:00 - 15:40	**Yuri Tomilov** *On C_0-semigroup with Riemann-Lebesgue property*	
	Parallel sessions	
	Session A (Ground floor) Chair: Cristina Câmara	**Session B (Upper floor)** Chair: Ernst Albrecht
15:45 - 16:05	**Marek Kosiek** *Some application of operator theory to analytic functions*	**Cristina Diogo** *Generalized inverses for a class of Toeplitz operators*
16:10 - 16:30	**Michał Wojtylak** *Domination in Krein spaces*	**Bojan Kuzma** *Quasi-commutativity preservers*
16:30 - 16:50	**Coffee break**	
16:50 - 17:10	**Wojciech Motyka** *Jacobi matrices with periodically modulated weights*	**Michal Zajac** *Hyperreflexivity of some operators*
17:15 - 17:35	**Michał A. Nowak** *Projection-iterative methods for a class of operator equations*	**Jan Vršovský** *Operators that are not orbit-reflexive*

Sunday, June 29th

8:00 - 20:00	**Excursion**

Monday, June 30th

Conference Hall: al. Mickiewicza 24/28

	Plenary session (Room 120) Chair: Mostafa Mbekhta
9:00 - 9:40	**Gustavo Corach** *Factorizations of projections and sampling theory in Hilbert spaces*
9:45 - 10:25	**Ameer Athavale** *Quasisimilarity-invariance of joint spectra for certain subnormal tuples*
10:30 - 11:10	**Vladimír Müller** *On smooth local resolvents*
11:10 - 11:30	**Coffee break**
	Chair: Aurelian Gheondea
11:30 - 12:10	**Michael Lin** *Ergodic theorems with rate for Dunford-Schwartz operators*
12:15 - 12:55	**Mostafa Mbekhta** *New results on linear preserver problems*
13:00 - 13:20	**Franciszek H. Szafraniec** *Subnormality and cyclicity: an open problem*
13:20 - 15:00	**Lunch break**
	Chair: Hari Bercovici
15:00 - 15:20	**Jan Janas** *Sharp bounds on the generalized eigenvectors of unbounded Jacobi operators*
15:25 - 15:45	**Janko Bračič** *Local spectra of a decomposable multiplication operator*
15:50 - 16:10	**Krzysztof Rudol** *Representing operators through frames*
16:15 - 16:35	**Zenon Jabłoński** *Operator moment problems*
16:35 - 23:00	**Conference dinner**

Tuesday, July 1st
Conference Hall: al. Mickiewicza 24/28

Plenary session (Room 120)	
	Chair: Martin Mathieu
9:00 - 9:40	**Jan Stochel** *Algebraic sets of type A, B and C coincide*
9:45 - 10:25	**Horst Behncke** *Projections and idempotents*
10:30 - 11:10	**László Kérchy** *Representation showing regular norm-behaviour*
11:10 - 11:30	**Coffee break**
	Chair: Gustavo Corach
11:30 - 11:50	**Mercedes Siles Molina** *Algebras of derivations that are strongly non-degenerate*
11:55 - 12:35	**Jaroslav Zemánek** *Quasinilpotent perturbations of the identity*
12:40 - 13:20	**Martin Mathieu** *Spectral isometries between C^*-algebras*

List of Participants

ALBRECHT, Ernst
Department of Mathematics, Saarland University
Postfach 15 11 50 , 66041 Saarbruecken, Germany
e-mail: `ernstalb@math.uni-sb.de`

ATHAVALE, Ameer
Department of Mathematics, Indian Institute of Technology Bombay
Powai, 400076 Mumbai, India
e-mail: `athavale@math.iitb.ac.in`

BEHNCKE, Horst
Department of Mathematics, University of Osnabrueck
Albrechtstr. 28 , D-49069 Osnabrueck, Germany
e-mail: `jones@mathematik.uni-osnabrueck.de`

BERCOVICI, Hari
Department of Mathematics, Indiana University
831 Rawles Hall, East 3rd St, Bloomington, IN 47405, USA
e-mail: `bercovic@indiana.edu`

BRAČIČ, Janko
University of Ljubljana, IMFM
Jadranska 19, SI-1000 Ljubljana, Slovenia
e-mail: `janko.bracic@fmf.uni-lj.si`

BUDZYŃSKI, Piotr
Department of Applied Mathematics, University of Agriculture in Krakow
al. Mickiewicza 24/28, 30-059 Krakow, Poland
e-mail: `piotr.budzynski@ar.krakow.pl`

BURDAK, Zbigniew
Department of Applied Mathematics, University of Agriculture in Krakow
al. Mickiewicza 24/28, 30-059 Krakow, Poland
e-mail: `rmburdak@cyf-kr.edu.pl`

CAMÂRA, Cristina
Department of Mathematics, Technical University of Lisbon
Av. Rovisco Pais, 1049-001 1049-001 Lisboa, Portugal
e-mail: `cristina.camara@math.ist.utl.pt`

CHMIELIŃSKI, Jacek
Institute of Mathematics, Pedagogical University of Cracow
ul. Podchorążych 2, 30-084 Krakow, Poland
e-mail: `jacek@ap.krakow.pl`

CICHOŃ, Dariusz
Institute of Mathematics, Jagiellonian University
ul. Reymonta 4, 30-059 Krakow, Poland
e-mail: `cichon@im.uj.edu.pl`

COJUHARI, Petru
Department of Applied Mathematics, AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
e-mail: cojuhari@uci.agh.edu.pl

CORACH, Gustavo
Instituto Argentino de Matemática (CONICET)
Saavedra 15, 1083 Buenos Aires, Argentina
and
Department of Mathematics, University of Buenos Aires
Paseo Colón 850, 1063 Buenos Aires, Argentina
e-mail: gcorach@gmail.com

DIOGO, Cristina
Departamento de Metodos Quantititivos, ISCTE
Av. das Forças Armadas, 1649-026 Lisbon, Portugal
e-mail: cristina.diogo@iscte.pt

GHEONDEA, Aurelian
Department of Mathematics, Bilkent University
06800 Bilkent, Ankara, Turkey,
e-mail: aurelian@fen.bilkent.edu.tr
and
Institute of Mathematics of the Romanian Academy
C.P. 1-764, 014700 Bucharest, Romania
e-mail: A.Gheondea@imar.ror

JABŁOŃSKI, Zenon
Institute of Mathematics, Jagiellonian University
ul. Reymonta 4, 30-059 Krakow, Poland
e-mail: jablonsk@im.uj.edu.pl

JANAS, Jan
Institute of Mathematics of the Polish Academy of Sciences
ul. Sw. Tomasza 30, 31-027 Krakow, Poland
e-mail: najanas@cyf-kr.edu.pl

JANG, Sun Young
Department of Mathematics, University of Ulsan
San 29, Moogeo-dong, Nam-gu, 680-749 Ulsan, South Korea
e-mail: jsym@mail.ulsan.ac.kr

KARMAKAR, Biswajit
Max Planck Institute for Mathematics in the Sciences
Inselstrasse 22 , D-0410 Leipzig, Germany
e-mail: karmakar@mis.mpg.de

KÉRCHY, László
Bolyai Institute, University of Szeged
Aradi vértanúk tere 1, 6720 Szeged, Hungary
e-mail: kerchy@math.u-szeged.hu

KLIŚ-GARLICKA, Kamila
Department of Applied Mathematics, University of Agriculture in Krakow
al. Mickiewicza 24/28, 30-059 Krakow, Poland
e-mail: rmklis@cyf-kr.edu.pl

KOSIEK, Marek
Institute of Mathematics, Jagiellonian University
ul. Reymonta 4, 30-059 Krakow, Poland
e-mail: Marek.Kosiek@im.uj.edu.pl

KULA, Anna
Institute of Mathematics, Jagiellonian University
ul. Reymonta 4, 30-059 Krakow, Poland
e-mail: Anna.Kula@im.uj.edu.pl

KUZMA, Bojan
Institute of Mathematics, Physics and Mechanics,
Jadranska 19, 1000 Ljubljana, Slovenia
and
Faculty of Education, University of Primorska
Cankarjeva 5, 6000 Koper, Slovenia
e-mail: bojan.kuzma@pef.upr.si

LIN, Michael
Department of Mathematics, Ben-Gurion University of the Negev
P.O.B. 653, Be'er-Sheva 84105, Israel
e-mail: lin@cs.bgu.ac.il

MATHIEU, Martin
Department of Pure Mathematics , Queen's University Belfast
Belfast BT7 1NN, Northern Ireland
e-mail: m.m@qub.ac.uk

MBEKHTA, Mostafa
UFR de Mathématiques Pures et Appliquées, Université de Lille 1
Cité Scientifique - Bât. M2, 59655 Villeneuve d'Ascq Cedex, France
e-mail: Mostafa.Mbekhta@math.univ-lille1.fr

MŁOCEK, Wojciech
Department of Applied Mathematics, University of Agriculture in Krakow
al. Mickiewicza 24/28, 30-059 Krakow, Poland
e-mail: wmlocek@ar.krakow.pl

MOTYKA, Wojciech
Institute of Mathematics of the Polish Academy of Sciences
ul. Sw. Tomasza 30, 31-027 Krakow, Poland
e-mail: namotyka@cyf-kr.edu.pl

MÜLLER, Vladimír
Mathematical Institute, Czech Academy of Sciences
Zitna 25, 115 67 Praha 1, Czech Republic
e-mail: muller@math.cas.cz

NOWAK, Michał A.
Department of Applied Mathematics, AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
e-mail: manowak@wms.mat.agh.edu.pl

PIWOWARCZYK, Kamila
Department of Applied Mathematics, University of Agriculture in Krakow
al. Mickiewicza 24/28, 30-059 Krakow, Poland
e-mail: kpiwowarczyk@ar.krakow.pl

PŁANETA, Artur
Institute of Mathematics, Jagiellonian University
ul. Reymonta 4, 30-059 Krakow, Poland
e-mail: Artur.Planeta@im.uj.edu.pl

PTAK, Marek
Department of Applied Mathematics, University of Agriculture in Krakow
al. Mickiewicza 24/28, 30-059 Krakow, Poland
e-mail: rmptak@cyf-kr.edu.pl

RUDOL, Krzysztof
Department of Applied Mathematics, AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
e-mail: grrudol@cyf-kr.edu.pl

SILES MOLINA, Mercedes
Department of Algebra, Geometry and Topology, University of Malaga
Campus de Teatinos PC: 29071, Malaga, Spain
e-mail: msilesm@uma.es

STOCHEL, Jan
Institute of Mathematics, Jagiellonian University
ul. Reymonta 4, 30-059 Krakow, Poland
e-mail: Jan.Stochel@im.uj.edu.pl

STRZĄDAŁA, Beata
Department of Applied Mathematics, AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
e-mail: beata.strzadala@gmail.com

SZAFRANIEC, Franciszek Hugon
Institute of Mathematics, Jagiellonian University
ul. Reymonta 4, 30-059 Krakow, Poland
e-mail: `Franciszek.Szafraniec@im.uj.edu.pl`

SZLACHTOWSKA, Ewa
Department of Applied Mathematics, AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
e-mail: `szlachto@wms.mat.agh.edu.pl`

TOMILOV, Yuri
Faculty of Mathematics and Informatics, Nicolaus Copernicus University
ul. Chopina 12/18, 87-100 Torun, Poland
e-mail: `tomilov@mat.uni.torun.pl`

VRŠOVSKÝ, Jan
Mathematical Institute, Czech Academy of Sciences
Zitna 25, 115 67 Praha 1, Czech Republic
e-mail: `vrsovsky@karlin.mff.cuni.cz`

WOJTYLAK, Michał
Department of Mathematics, VU University Amsterdam
De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands
e-mail: `Michal.Wojtylak@gmail.com`

WYSOCZAŃSKI, Janusz
Mathematical Institute, Wroclaw University
pl. Grunwaldzki 2/4, 50-384 Wroclaw, Poland
e-mail: `jwys@math.uni.wroc.pl`

ZAJAC, Michal
Department of Mathematics FEI, Slovak University of Technology in Bratislava
Ilkovicova 3, SK-812 19 Bratislava, Slovak Republic
e-mail: `michal.zajac@stuba.sk`

ZALOT, Ewelina
Department of Applied Mathematics, AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
e-mail: `ewelina.zalot@gmail.com`

ZEMÁNEK, Jaroslav
Institute of Mathematics of the Polish Academy of Sciences
P.O. Box 21, 00-956 Warsaw, Poland
e-mail: `zemanek@impan.gov.pl`

ZYGMUNT, Marcin
Department of Applied Mathematics, AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Krakow, Poland
e-mail: `marcin.zygmunt@vp.pl`

Abstracts

Chebyshev polynomials for operators

Ernst Albrecht

In 1971 Paul Halmos introduced Chebysheff polynomials for elements of a normed algebra $\mathcal{R}$: a monic polynomial p_n of degree n is called a *Chebyshev polynomial of order* n *for an element* $x \in \mathcal{R}$ if

$$\|p_n(x)\| = \inf_{p \in \mathcal{P}_n^1} \|p(x)\|,$$

where $\mathcal{P}_n^1$ is the set of all moni polynomials of degree n.

In this talk we investigate the problem of uniqueness uniqueness of Chebysheff polynomials of order n for bounded linear operators an normed spaces.

Quasisimilarity-invariance of joint spectra for certain subnormal tuples

Ameer Athavale

We investigate the invariance of the joint Taylor spectrum and the joint essential Taylor spectrum under quasisimilarity in the context of a special class of subnormal operator tuples associated with the open unit ball $\mathbb{B}^{2m}$ in $\mathbb{C}^m$. We show in particular that a subnormal m-tuple that is quasisimlar to either the Szegö tuple or the Bergman tuple has its Taylor spectrum equal to the closure $\overline{\mathbb{B}^{2m}}$ of $\mathbb{B}^{2m}$ and its essential Taylor spectrum equal to the unit sphere $\mathbb{S}^{2m-1}$, the topological boundary of $\mathbb{B}^{2m}$. A parallel investigation goes through for a class of subnormal tuples associated with the open unit polydisk $\mathbb{D}^m$ in $\mathbb{C}^m$.

The extreme points of the Horn body

Hari Bercovici

Given nonnegative complex matrices A, B, C such that $A + B + C = I$, we denote by $E(A, B, C)$ the $3n$-tuple consisting of the eigenvalues of these matrices. Thus, the first n entries are the eigenvalues of A in nondecreasing order and repeated according to multiplicity, the second n entries correspond to B, and the last n to C. The collection $H(n)$ of all $3n$-tuples obtained this way is now known to be a convex polyhedron. We will describe the elements of $H(n)$ in terms of certain measures on the plane. This allows us to find the extreme points of this polyhedron.

Local spectra of a decomposable multiplication operator

Janko Bračič

Let A be a unital semisimple commutative complex Banach algebra. If $a \in A$ induces a decomposable multiplication operator T_a on A, then the local spectra of T_a are given by $\sigma_{T_a}(x) = \widehat{a}(supp\,\widehat{x})$ $(x \in A)$. We show that the support $supp\,\widehat{x}$ can be replaced by the Beurling spectrum $sp(x)$ of $x \in A$. Then we derive the formula $\sigma_{T_a}(x) = \widehat{a}(sp(x))$ $(x \in A)$ also for some $a \in A$ when semisimplicity is not assumed.

Almost periodic factorization for a class of triangular matrix symbols

Cristina Câmara

Joint work with Yu. I. Karlovich and I. M. Spitkovsky.

Almost periodic (AP) factorization of matrix functions appears as a natural and important step in the study of convolution-type operators and Toeplitz operators with almost periodic symbols. In particular, when considering convolution operators on a finite interval of lenght λ, we are lead to the study of 2×2 matrix symbols of the form

$$G = \begin{bmatrix} e_\lambda & 0 \\ f & e_{-\lambda} \end{bmatrix} \tag{1}$$

where e_λ denotes the function defined by $e_\lambda(\xi) = e^{i\lambda\xi}$, $\xi \in R$, and f is an almost periodic function. Despite the apparently simple structure of these matrices, factorability criteria are known only under some rather restrictive additional conditions on f and the same applies to explicit formulas for the factors. Determining conditions for the AP factorization to be canonical, i.e., for the so called partial AP indices to be equal to zero, is particularly important since it corresponds to invertibility of the Toeplitz operator with symbol G.

In this talk some new cases are presented for which factorability criteria and formulas for the partial AP indices are obtained and the factorization itself can be constructed explicitly. Some a priori conditions on the Bohr-Fourier spectra of the factors are also given, provided that a canonical factorization exists.

Spectral analysis of an operator pencil with applications to linear transport

Petru A. Cojuhari

The mathematical equation describing the transport of energy through a medium is an integro-differential equation of the form [1], [2], [3]:

$$\mu \frac{\partial u(x,\mu)}{\partial x} + u(x,\mu) = \int_{-1}^{1} k(\mu,\mu^{'}) u(x,\mu^{'}) d\mu^{'}, \quad o \leq x < \infty, \tag{2}$$

under the boundary conditions

$$\lim_{x \longrightarrow \infty} u(x,\mu)\exp(x/\mu) = 0, \quad -1 \leq \mu < 0, \quad u(0,\mu) = f_+(\mu), \quad 0 \leq \mu \leq 1, \tag{3}$$

where $f_+(\mu)$, $0 \leq \mu \leq 1$ is a square integrable function. The kernel $k(\mu,\mu^{'})$ is a given real-valued and symmetric L_1-function on $-1 \leq \mu,\mu^{'} \leq 1$.

The problem (1) - (2) leads to the study of spectral properties of a linear operator pencil of the form $L(\lambda) = I - \lambda A + C$, where A is the operator of multiplication by the independent variable and C is an integral operator defined by the kernel $k(\mu,\mu^{'})$.

Our purpose is to study the spectrum of the operator pencil $L(\lambda)$. In particular, estimates for the point spectrum are given.

References

[1] K. M. Case, P. F. Zweifel, *Linear Transport Theory,* Reading, Mass., Addison-Wesley, 1967.
[2] B. Davison, *Neutron Transport Theory,* Oxford, 1960.
[3] M. V. Maslennikov, *The Milne problem with anisotropic-scattering,* Proc. Steklov Inst., 97 (1968) (Russian).

Factorizations of projections and sampling theory in Hilbert spaces

Gustavo Corach

There is a close connection between sampling formulae in reproducing kernel Hilbert spaces and projection on these spaces. We show how to minimize the norm of $F - H$, where Q is a fixed projection, $Q = FH^*$, and F and Q have the same range and the range of H is the orthogonal complement of the kernel of Q.

Generalized inverses for a class of Toeplitz operators

Cristina Diogo

Joint work with M. C. Câmara.

The invertibility and Fredholmness of a Toeplitz operator with 2×2 matrix symbol G, T_G, can be studied in connection with some properties of a solution to a Riemann-Hilbert problem with coefficient G. We show that from this solution we can obtain formulas for the factors of a Wiener-Hopf factorization of G, when it exists, which allows us to construct a generalized inverse for the operator T_G.

Absolute continuity of operator valued completely positive maps

Aurelian Gheondea

Joint work with Ali Şamil Kavruk

We investigate the notion of absolute continuity and Lebesgue-Radon-Nikodym type decompositions for operator valued completely positive maps on C^*-algebras, following the investigation of K.R. Parthasarathy in 1996. We first show that the construction of K.R. Parthsarathy produces actually the maximal Lebesgue decomposition, such that the maximality property makes it unique, and in which absolute continuity and singularity are exactly the natural definitions that we employ. Then we show that one can find a framework in which the maximal Lebesgue decomposition does not depend on the dominating completely positive map and hence, that more flexible formulae become available. We also illustrate the applicability of the Radon-Nikodym derivative and Lebesgue type decomposition to the infimum problem for completely positive maps, and then to known special similarity problems for operator valued completely bounded maps on C^*-algebras. In the end we specialize to completely positive maps in matrices and indicate how to calculate the Radon-Nikodym derivatives and Lebesgue decompositions in computable terms, that are of interest for applications to quantum information theory. Here a key role is played by the tracial completely positive map that is maximal in a certain sense.

Operator moment problems

Jabłoński Zenon

Let $\mathcal{D}$ be a complex inner product spaces and let $\mathcal{H}$ be a complex Hilbert space. We denote by $\boldsymbol{L}(\mathcal{D},\mathcal{H})$ (resp. $\boldsymbol{B}(\mathcal{H})$) the set of all linear (resp. bounded linear) operators from $\mathcal{D}$ to $\mathcal{H}$ (resp. from $\mathcal{H}$ to $\mathcal{H}$). An operator moment problem entails determining whether, for a given family $\{A^x_{\boldsymbol{m}}\}_{\substack{\boldsymbol{m}\in\mathbb{Z}^d_+\\ x\in X}} \subset \boldsymbol{L}(\mathcal{D},\mathcal{H})$ of operators, there exists a d-tuple $\boldsymbol{T}=(T_1,\ldots,T_d)\in\boldsymbol{B}(\mathcal{H})^d$ (with some additional properties) such that

$$A^x_{\boldsymbol{m}} = \boldsymbol{T}^{\boldsymbol{m}} A^x_{\boldsymbol{0}}, \quad \boldsymbol{m}\in\mathbb{Z}^d_+,\ x\in X.$$

The lecture will be a survey of results concerning an operator moment problem.

References

[1] Z. J. Jabłoński, J. Stochel, F. H. Szafraniec, Unitary propagation of operator data, *Proc. Edinb. Math. Soc.* (2) 50 (2007), 689-699.

[2] Z. J. Jabłoński, Moment problem with contractive solutions - the regular case, *Proc. Amer. Math. Soc.* 35 (2007), 2811-2819

[3] Z. J. Jabłoński, Moment theorem for completely hyperexpansive operators, *to appear in* Acta Math. Hungar.

Generalized Toeplitz algebra of strongly perforated semigroup

Sun Young Jang

When G is a discrete group and M is a subsemigroup of G, the Wiener-Hopf C^*-algebra $W(G,M)$ is the C^*-algebra generated by the left regular isometric representation of M. If the C^*-algebra generated by isometric representation of semigroups have the uniqueness property , the structure of those C^*-algebras are to some extent independent of the choice of isometries on a Hilbert space. Toeplitz algebra, the Cuntz algebra, and the C^*-algebra generated by one-parameter semigroups of isometries studied by R. Douglas are the outstanding examples of C^*-algebras with the uniqueness property. We show that the unperforated property of partially ordered abelian semigroups have a deep relationship with the uniqueness property of C^*-algebras. And also we show that $W(\mathbf{Z},P)$ is isomorphic to the Toeplitz algebra and $W(\mathbf{Z},P)$ doesn't have the uniqueness property for a strongly perforated subsemigroup P of the integer group $\mathbf{Z}$.

Representation showing regular norm-behaviour

László Kérchy

Let $\rho\colon S\to\mathcal{L}(\mathcal{X})$ be a strongly continuous representation of the locally compact, abelian semigroup S on the Banach space $\mathcal{X}$. Regular norm-behaviour of ρ means the existence of a function $p\colon S\to\mathbb{R}_+$ with nice properties, dominating the norm-function of ρ. We give a survey on our results connected with such representations. Characterizations of regularity are provided in the particular cases, when S is $\mathbb{Z}_+$ or $\mathbb{R}_+$.

Some application of operator theory to analytic functions

Marek Kosiek

It is shown that for each open subarc E of the unit circle there exists a function of class H^1_0 with nonnegative real part and zero imaginary part on E.

Quasi-commutativity preservers

Bojan Kuzma

Joint work with G. Dolinar

Let M_n be the algebra of all $n \times n$ matrices over $\mathbb{C}$. We say that $A, B \in M_n$ quasi-commute if AB and BA are linearly dependent and either $AB = 0 = BA$ or they are both nonzero. This is equivalent to the existence of a nonzero $\xi = \xi_{A,B} \in \mathbb{C}$ such that $AB = \xi BA$. The relation of quasi-commutativity has applications in quantum mechanics. Recently, there has been quite some activity in classifying the linear maps, which preserve this relation. We follow the current study of nonlinear preservers and classify bijective, not necessarily linear, maps $\Phi\colon M_n \to M_n$ which preserve quasi-commutativity in both directions. By avoiding the assumptions of linearity, we thus classified isomorphisms of the structure on M_n, given by the quasi-commutativity relation. The results are close to commutativity-preservers but differ slightly from them.

Ergodic theorems with rate for Dunford-Schwartz operators

Michael Lin

Let T be a a mean ergodic contraction on a Banach space X, i.e., such that $\frac{1}{n}\sum_{k=1}^{n} T^k x$ converges in norm for every $x \in X$. Unless $(I-T)X$ is closed, there is no general rate in the mean ergodic theorem. We will consider the rate of convergence $\|\frac{1}{n}\sum_{k=1}^{n} T^k x\| = \mathcal{O}(1/n^\alpha)$ for $\alpha \in (0,1)$.

A particular case of interest in the mean ergodic theorem is that of Dunford-Schwartz operators, which are contractions of $L_1(m)$ of a probability space which are also contractions of L_∞, and therefore also of all L_p, $1 < p < \infty$. For T a Dunford-Schwartz contraction there is also a pointwise ergodic theorem – the averages $\frac{1}{n}\sum_{k=1}^{n} T^k f$ converge a.e. for any $f \in L_1$. Again, there is no uniform rate of the a.e. convergence.

Our purpose is to obtain growth conditions on $\|\sum_{k=1}^{n} T^k f\|_p$ which ensure that $\frac{1}{n^{1/p}}\sum_{k=1}^{n} T^k f$ converges to 0 *almost everywhere*. We will explain in what sense for $f \in L_p$ the rate $n^{1/p}$ is critical. This is connected to *norm convergence* of series of the type $\sum_{k=1}^{\infty} \frac{T^k f}{k^{1-\alpha}}$ for some $\alpha \in (0,1)$.

Spectral isometries between C^*-algebras

Martin Mathieu

In this talk I plan to survey some recent results on the structure of spectral radius-preserving linear mappings between C^*-algebras. These are related to Kadison's 1951 result on the structure of isometries on C^*-algebras as well as Kaplansky's 1970 question on invertibility-preserving operators.

New results on linear preserver problems

Mostafa Mbekhta

Let H be an infinite-dimensional complex separable Hilbert space and $\mathcal{B}(H)$ the algebra of all bounded linear operators on H. In this talk, we discuss the following new results:

Theorem I Let H be an infinite-dimensional separable Hilbert space and $\phi\colon \mathcal{B}(H) \to \mathcal{B}(H)$ a linear map preserving generalized invertibility in both directions. Assume that ϕ is surjective up to finite rank operators. Then

$$\phi(\mathcal{K}(H)) \subseteq \mathcal{K}(H)$$

and there exist an invertible element $\mathbf{a} \in \mathcal{C}(H)$ and either an automorphism $\tau\colon \mathcal{C}(H) \to \mathcal{C}(H)$ or an anti-automorphism $\tau\colon \mathcal{C}(H) \to \mathcal{C}(H)$ such that the induced map $\varphi\colon \mathcal{C}(H) \to \mathcal{C}(H)$, $\varphi(A + \mathcal{K}(H)) = \phi(A) + \mathcal{K}(H)$, $A \in \mathcal{B}(H)$, is of the form

$$\varphi(x) = \mathbf{a}\tau(x), \quad x \in \mathcal{C}(H).$$

Theorem II Let H be an infinite-dimensional separable Hilbert space and $\phi\colon \mathcal{B}(H) \to \mathcal{B}(H)$ a linear map preserving semi-Fredholm operators in both directions. Assume that ϕ is surjective up to compact operators. Then

$$\phi(\mathcal{K}(H)) \subseteq \mathcal{K}(H)$$

and the induced map $\varphi\colon \mathcal{C}(H) \to \mathcal{C}(H)$ is either an automorphism, or an anti-automorphism multiplied by an invertible element $\mathbf{a} \in \mathcal{C}(H)$.

Theorem III Under the same hypothesis and notation as in (II), the following statements hold true:

(i) ϕ preserves Fredholm operators in both directions;

(ii) there is an $n \in Z$ such that either

$$\mathrm{ind}(\phi(T)) = n + \mathrm{ind}(T) \text{ or } \mathrm{ind}(\phi(T)) = n - \mathrm{ind}(T)$$

for every Fredholm operator T.

Observe that, every $n \times n$ complex matrix has a generalized inverse (resp. is semi-Fredholm, Fredholm), and therefore, every linear map on a matrix algebra preserves generalized invertibility (resp. semi-Fredholm, Fredholm) in both directions. So, we have here an example of a linear preserver problem which makes sense only in the infinite-dimensional case.

References

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Jacobi matrices with periodically modulated weights

Wojciech Motyka

We deal with a class of Jacobi matrices defined by: $\lambda_n = c_n n^\alpha$ and $q_n = b_n n^\alpha$, where $\alpha \in (0,1)$, $n \in \mathbb{N}$, (c_n) and (b_n) are real non-zero two-periodic sequences such that $c_1,\ c_2 > 0$ and $b_1 \neq -b_2$. We are interested in spectral properties of such operators which are in the double root case, namely $|b_1 b_2 - c_1^2 - c_2^2| = 2c_1 c_2$. Using W.Kelley's approach (in the non-oscillatory situation) or the ansatz approach (in the oscillatory situation) we will give asymptotic formulas of solutions of the generalized eigenequation associated with the corresponding Jacobi operator $\mathcal{J}$. From those formulas, via the subordination theory and an estimation of the scalar product of two eigenvectors, we will obtain discreetness of the spectrum of $\mathcal{J}$ in one halfline of the real axis (positive or negative, depending on the parameters b_1, b_2, c_1, c_2) and absolute continuity in the other one.

On smooth local resolvents

Vladimír Müller

Let T be a bounded linear operator on a Banach space X. It is well-known that the resolvent $z \mapsto (T-z)^{-1}$ is always unbounded near the spectrum of T. Recently it was observed by M. Gonzalez that the local resolvent $z \mapsto (T-z)^{-1}x$ may be bounded (for some fixed $x \in X$). This phenomenon was later investigated by several authors. We will show the local resolvent may be smooth. More precisely, there are an operator T, vector x and an everywhere defined C^∞-function $f : \mathbf{C} \to X$ such that $(T-z)f(z) = x$ for all z.

Projection-iterative methods for a class of operator equations

Michał A. Nowak

Joint work with P. A. Cojuhari

Projection-iterative methods are proposed for approximation solutions of operator equations of the form

$$Au + Bu = f\,.$$

where A and B are linear bounded operators in a Banach space $\mathbb{E}$ and f is a given element in $\mathbb{E}$. Applications to concrete classes of equations are presented. In particular, Wiener-Hopf type equations and their discrete analogues are studied.

Representation of some classes of operators via frames

Krzysztof Rudol

Some consequences of infinite matrix representation of certain operators in overcomplete systems in a Hilbert space are given. Certain properties of expansions in orthonormal bases are preserved, like membership criteria in Schatten - von Neumann classes, but some problems arise as well. Perspectives for analysis of the spectra from these representations will also be discussed.

Algebras of derivations that are strongly non-degenerate

Mercedes Siles Molina

A Lie algebra L is said to be strongly non-degenerate (following the terminology by Kostrikin) if $[x,[x,L]]=0$ for some x in L implies $x=0$. We will see that the Lie algebra $Der(A)$ consisting of associative derivations of a semiprime non-commutative associative algebra A is strongly non-degenerate whenever the center of A does not contain non-zero associative ideals (this happens, for example, whenever A is prime). Our result extends that by Jordan and Jordan in 1978, where they showed that if A is a prime associative non-commutative algebra, the the Lie algebra $Der(A)$ is prime.

The key point is a theorem that describes the quadratic annihilator of a Lie algebra inside appropriate Lie overalgebras. We give other applications of this result in the context of algebras of quotients of Lie algebras, notion introduced by the speaker that will be explored in this talk.

Algebraic sets of type A, B and C coincide

Jan Stochel

It is proved that the definition of an algebraic set of type A (a notion related to the multidimensional Hamburger moment problem) does not depend on the choice of a polynomial describing the algebraic set in question and that an algebraic set of type B is always of type A. This answers in the affirmative two questions posed in 1992 by the author. It is also shown that an algebraic set is of type A if and only if it is of type C (a notion linked to orthogonality of polynomials of several variables). This, in turn, enables us to answer three questions posed in 2005 by Cichoń, Stochel, and Szafraniec.

Subnormality and cyclicity: an open problem

Franciszek H. Szafraniec

I intend to introduce (some in) the audience into the problem.

On C_0-semigroup with Riemann-Lebesgue property

Yuri Tomilov

Joint work with R. Chill

We will discuss a new method for constructing C_0-semigroups for which properties of the resolvent of the generator and continuity properties of the semigroup in the operator-norm topology are controlled simultaneously. The method allows one to construct various examples ruling out any possibility of characterizing norm-continuity of semigroups on arbitrary Banach spaces in terms of resolvent-norm decay on vertical lines. In particular, it leads to a solution of the so-called "norm-continuity problem" for semigroups posed by A. Pazy in 1983. (Another solution of the problem was found recently by T. Matrai.)

Operators that are not orbit-reflexive

Jan Vršovský

Joint work with Vladimír Müller

Let T be a bounded linear operator on a (real, complex) Banach space X. Analogously to the definition of reflexivity, we say that T is orbit-reflexive if every bounded linear operator A belongs to the closure of $\{T^n : n = 1, 2, 3 \ldots\}$ in the strong operator topology whenever $Au \in \overline{\{T^n u : n = 1, 2, 3 \ldots\}}$ for each $u \in X$. While the notion of reflexivity is connected to the problem of invariant subspaces, orbit-reflexivity is in the same way connected to the problem of invariant subsets.

Recently, Sophie Grivaux and Maria Roginskaya found a Hilbert space operator which is not orbit-reflexive. We present a different, more simple type of construction that also provides a Hilbert space operator which is not orbit-reflexive, and moreover a Banach space operator which is reflexive but not orbit-reflexive.

Domination in Krein spaces

Michał Wojtylak

We provide two criteria for selfadjointness in a Krein space, which are closely connected with the notion of domination of unbounded operators. Recall, that an operator A is said to *dominate* B *on the space* $\mathcal{E} \subseteq \mathcal{D}(A) \cap \mathcal{D}(B)$ if

$$\|Bf\| \leq c(\|f\| + \|Af\|), \qquad f \in \mathcal{E},$$

for some constant $c \geq 0$. The second important concept in the talk is the space of bounded vectors of an operator, defined as the union (taken over all $a > 0$) of the sets

$$\mathcal{B}_a(A) := \{f \in \mathcal{D}^\infty(A) : \exists c > 0 \quad \forall n \in \mathbb{N} \quad \|A^n f\| \leq ca^n\}.$$

First of the results ([2]) says the following. *If A is a symmetric operator in a Krein space $\mathcal{K}$ and there exists a sequence $(T_n)_{n=0}^\infty \subseteq \mathbf{B}(\mathcal{K})$ such that*

- $T_n \to I$ *in WOT,*
- $T_n(\mathcal{K}) \cup T_n^+(\mathcal{K}) \subseteq \mathcal{D}(A)$ *for* $n \in \mathbb{N}$,
- $\sup_{n\in\mathbb{N}} \|AT_n - T_nA\| < \infty$,

then A is selfadjoint. As an example of the operators T_n ($n \in \mathbb{N}$) one can take the projections onto the spaces $\mathcal{B}_n(S)$ of some selfadjoint operator S.

In the second theorem ([3]) it is assumed that a system of pointwise commuting symmetric operators in a Pontriagin space is given. Moreover, one of them is selfadjoint and dominates all the others. It appears that in such a situation all these operators are selfadjoint and that they commute spectrally. The result is an indefinite inner product version of a theorem from [1]. In both papers the bounded vectors turned up as a very useful tool in the proof.

References

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Hyperreflexivity of some operators

Michal Zajac

In the talk the question for which inner function $m(\lambda)$ there exists a C_0 contraction T with hyperreflexive commutant will be considered. C_0 contraction having reflexive commutant were characterized in 1992 by V.V. Kapustin. In particular, if the minimal function $m(\lambda)$ is a Blaschke product with simple zeros, then its commutant $\{T\}'$ is reflexive. We show that for such a Blaschke product $B(\lambda)$ there exists a C_0 contraction T the minimal function of which is $B(\lambda)$ and its commutant is hyperreflexive with constant od hyperreflexivity $\kappa_{\{T\}'} = 1$. The author was partially supported by grant G-1/3025/06 of MŠ SR

Quasinilpotent perturbations of the identity

Jaroslav Zemánek

Let Q be a quasinilpotent operator. Some years ago, Olavi Nevanlinna suggested to study the set of complex numbers z such that the operator $I - zQ$ satisfies certain resolvent or geometric properties (like the Kreiss condition, power boundedness, etc.). We intend to report on the current research including, in particular, recent results obtained jointly with Alexander Gomilko.