Semi-hyponormality of commuting pairs of Hilbert space operators

Raúl E. Curto (The University of Iowa, Iowa City, Iowa, United States)

We will first describe an explicit formula for the square root of positive 2×2 operator matrices with commuting entries, and then use it to define and study semi-hyponormality for commuting pairs of Hilbert space operators. For the well-known 3–parameter family $W_{(\alpha,\beta)}(a, x, y)$ of 2–variable weighted shifts, we have been able to completely identify the parametric regions in the open unit cube where $W_{(\alpha,\beta)}(a, x, y)$ is subnormal, hyponormal, semi-hyponormal, and weakly hyponormal. As a result, we describe in detail concrete sub-regions where, for instance, weak hyponormality holds but semi-hyponormality does not hold. To accomplish this, we employ a new technique emanating from the homogeneous orthogonal decomposition of $\ell^2(\mathbb{Z}_+^2)$.

The talk is based on joint work with Jasang Yoon (The University of Texas Rio Grande Valley, United States).