Strictly stable Hurwitz polynomials and their determinantal representations

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We consider complex polynomials that are stable, i.e., have no zeroes, on a Siegel domain of the first kind. Such polynomials are closely related to real homogeneous polynomials that are hyperbolic and to their hyperbolicity cones, that play a central role in convex algebraic geometry. Motivated by the generalized Lax conjecture that any hyperbolicity cone admits a linear matrix inequality representation, we consider a certifying linear determinantal representation for complex stable polynomials. We establish the existence of such a certifying determinantal representation, up to a cofactor, for the case of Siegel domains of the first kind over a symmetric homogeneous cone, such as the positive orthant or more generally a product of (symmetric or skew-symmetric) matrix halfspaces, and for polynomials that are strictly stable, i.e., have no zeroes on the boundary of the domain and do not vanish, in an appropriate sense, at infinity. The proofs use a Cayley transform and certifying determinantal representations for strictly stable polynomials on a conformally equivalent bounded (symmetric) domain, using a Hermitian Positivstellensatz and a lurking contraction argument generalizing earlier work in [Grinshpan et. al., *Oper. Th. Adv. Appl.*, 255 (2016), 123–136].

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