## Spectral set, complete spectral set and dilation for Banach space operators

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Famous results due to von Neumann, Sz.-Nagy and Arveson assert that the following four statements are equivalent; a Hilbert space operator $T$ is a contraction; the closed unit disk $\overline{\mathbb{D}}$ is a spectral set for $T$; $T$ can be dilated to a Hilbert space isometry; $\overline{\mathbb{D}}$ is a complete spectral set for $T$. In this talk, we show by counterexamples that no two of them are equivalent for Banach space operators. If $\mathcal{F}_r$ is the family of all complex Banach space operators having norm less than or equal to $r$ and if $D_R$ denotes the open disk in the complex plane with centre at the origin and radius $R$, then we show by an application of Bohr's theorem that $\overline{D}_R$ is the minimal spectral set for $\mathcal{F}_r$ if and only if $r = R/3$. For a complex Banach space $\mathbb{X}$, we show that the following statements are equivalent: $(i)$ $\mathbb{X}$ is a Hilbert space; $(ii)$ $\overline{\mathbb{D}}$ is a spectral set for the forward shift operator $M_z$ on $\ell_2(\mathbb{X})$; $(iii)$ $\overline{\mathbb{D}}$ is a spectral set for the backward shift operator $\widehat{M_z}$ on $\ell_2(\mathbb{X})$; $(iv)$ $\overline{\mathbb{D}}$ is a spectral set for every strict contraction on $\mathbb{X}$; $(v)$ $\overline{\mathbb{D}}$ is a complete spectral set for every contraction $T$ on $\mathbb{X}$ with $\|T\| = 1$; $(vi)$ $\overline{\mathbb{D}}$ is a complete spectral set of the identity operator $I_{\mathbb{X}}$ on $\mathbb{X}$.

This is a joint work with Swapan Jana.

[1] S. Jana and S. Pal, Spectral set, complete spectral set and dilation for Banach space operators, *Preprint, https://arxiv.org/abs/2411.01605.*