A Hardy space on the unit cotangent bundle of the sphere and its operators

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Let us consider the unit cotangent bundle $T_1^*S^n \subset \mathbb{R}^{2n+2}$ of the sphere $S^n \subset \mathbb{R}^{n+1}$, both endowed with the inherited Riemannian metrics. The compact Riemannian manifold $T_1^*S^n$ admits an isometric action of $\mathbb{T} \times O(n+1)$, the product of the unit circle $\mathbb{T}$ and the group $O(n+1)$ consisting of the orthogonal matrices with size $(n+1) \times (n+1)$. The latter action comes from the isometric action on S^n and the former is given by the geodesic flow. By the classical construction of Grauert tubes, there is a domain Ω of a cone $C \subset \mathbb{C}^{n+1}$ such that the boundary X of Ω is canonically identified with $T_1^*S^n$, and also so that the $\mathbb{T} \times O(n+1)$-action carries over naturally to X. This construction allows to introduce a Hardy space $H(X)$ that can be related to function spaces on $T_1^*S^n$. In this talk we will explain how this construction allows to study operators, most importantly pseudo-differential ones, on S^n through the Hardy space $H(X)$. This is particularly useful in the case of operators that intertwine either of the actions of $\mathbb{T}$ or $O(n+1)$.