A mathematical study of multi-spatial and multi-fractional damped nonlinear wave models

Pratibha Verma (Poznan University of Technology, Poznań, Poland)

This research investigates a multi-fractional nonlinear generalized diffusion wave equation involving a multi-spatial Riesz operator and a damping term, analyzed through fractional operators. In this study, we employ the fractional Laplace operator in symbolic form. This operator possesses various definitions and plays a crucial role in dynamic systems, especially in bounded domains, where it effectively handles boundary conditions. We propose a non-linear generalized damped wave equation with homogeneous boundary conditions, utilizing the Riesz fractional derivative in one dimension (1D) for converting the fractional Laplace operator into the Riesz fractional operator. The Caputo-Fabrizio derivative is incorporated to capture the damping effects and explore the behavior of the nonlinear generalized diffusion-wave equation. The primary objective of this study is to examine the Hyers-Ulam stability of the proposed multi-fractional model, establishing that small perturbations in the system do not result in divergence. This analysis leads to robust results for fractional systems, offering highly accurate approximations of the system solution. Several examples are presented in one, two, and three spatial dimensions to validate the theoretical results. Graphical illustrations depict the evolution of the error in the spatial and temporal domains, demonstrating that the error remains bounded. The 2D and 3D plots further confirm that the system stabilizes under perturbations, preventing instability. Furthermore, Hyers-Ulam stability is rigorously justified for various fractional orders, reinforcing the reliability of the results.

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