When is a CPD operator similar to a subnormal operator?

Jan Stochel (Jagiellonian University, Kraków, Poland)

An operator T on a Hilbert space H is said to be *conditionally positive definite (CPD)* if, for every $f \in H$, the sequence $\{\|T^n f\|^2\}_{n=0}^{\infty}$ is conditionally positive definite on the semigroup $(\mathbb{Z}_+, +)$, where $\mathbb{Z}_+$ denotes the set of all nonnegative integers. According to the celebrated Lambert's theorem, T is subnormal (i.e., the restriction of a normal operator to a closed invariant subspace) if and only if, for every $f \in H$, the sequence $\{\|T^n f\|^2\}_{n=0}^{\infty}$ is positive definite on $(\mathbb{Z}_+, +)$.

We show that a CPD unilateral weighted shift W_λ of type III is a quasi-affine transform of the operator M_z, representing multiplication by the independent variable z on the $L^2(\rho)$-closure of analytic complex polynomials on the complex plane, where ρ is a measure precisely determined by W_λ. We completely characterize when a CPD unilateral weighted shift is similar to a subnormal operator. This is also done for operator-weighted shifts. Additionally, necessary and separately sufficient conditions for a general CPD operator to be similar to a subnormal one are provided.