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Abstracts

Local commutants of operators on Banach spaces

Janko Bračič

Joint work with Vladimir Müller

Let X be a complex Banach space and let $B(X)$ be the algebra of all bounded linear operators on X. The local commutant of $T \in B(X)$ at $x \in X$ is

$$C(T, x) = \{S \in B(X); \quad (TS - ST)x = 0\}.$$

These spaces of operators were studied first by David R. Larson [2] ten years ago in the context of wavelets (see also Ch. 1 in [1]).

In the talk we present some properties of local commutants and we discuss about the reflexivity of these spaces. At the end a few open questions are mentioned.

References

[1] X. Dai and D. R. Larson, *Wandering Vectors for Unitary Systems and Orthogonal Wavelets*, Mem. Amer. Math. Soc., **134** No. 640 (1998). MR1432142 (98m:47067).

[2] D. R. Larson, *Von Neumann algebras and wavelets*, Operator algebras and applications (Samos, 1996), 267–312, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 495, Kluwer Acad. Publ., Dordrecht, 1997. MR1462685 (98g:46091)

The decomposition of pairs of commuting power partial isometries

Zbigniew Burdak

We are going to present the generalization of decomposition of a power partial isometry to a pair of such operators which commute but not necessarily doubly commute. The generalization of decompositions of certain class of operators to a pair of such operators causes some problems even in much better known class of operators (like the generalization of decomposition of isometry to a unitary operator and a unilateral shift - Wold). Recall that an operator is called *a power partial isometry* if every power of the operator is a partial isometry (f.i. is an isometry on the orthogonal complement of the kernel). Recall also that *a truncated shift of index* n ($n \in \mathbb{Z}$) is an operator T on a Hilbert space $H \times H \cdots \times H$($n-$times) given by formula $T\langle x_1, \ldots, x_n\rangle = \langle 0, x_1, \ldots, x_{n-1}\rangle$. The class of power partial isometries was considered by Wallen and Halmos. They showed that every power partial isometry can be decomposed into: a unitary operator, a unilateral shift, a backward shift and truncated shifts of positive integer indexes. There is a natural extension of that decomposition to a pair of doubly commuting power partial isometries (operators doubly commute when they commute and each of them commute with the adjoint of the other one). Such a pair of operators is decomposed to: a pair of unitary operators, a pair of a unitary operator and a unilateral shift, a pair of a unitary operator and a backward shift and other pairs of all possible combinations of operators decomposing power partial isometries. Assuming that power partial isometries only commute, we obtain much more complicated case. We can have for example a pair of power partial isometries which product is not a power partial isometry and other cases which we are going to describe.

Joint subnormality of operator valued functions

Dariusz Cichoń

We discuss the question of joint subnormality of analytic operator-valued functions on open subsets of normed spaces with special emphasis on the role played by sets of uniqueness in recovering the joint subnormality. Minimal normal extensions of such functions are characterized via coefficients of their Taylor series expansions. Investigating perturbations of unitary operators and subnormal partial isometries gives rise to numerous illustrative examples. We supply an explicit matrix construction of normal extensions of specific perturbations of subnormal partial isometries.

Spectral analysis of Hessenberg type matrices

Petru A. Cojuhari

Hessenberg type matrices appear in connection with the study of some problems from the theory of orthogonal polynomials. Namely, if μ is a given probability measure on the unit circle T, it can be defined a system of orthogonal polynomials $P_n(z) = c_n z^n + ... + c_0$ $(n = 0, 1, ...)$ on T by the requirement that $c_n > 0$, and that

$$\int_T P_n(z)\overline{P_m(z)}d\mu = \delta_{nm} \ (n, m = 0, 1, ...).$$

Let $\widetilde{P}_n(z) = c_n^{-1} P_n(z)$ $(n = 0, 1, ...)$ be the corresponding monic orthogonal polynomials, and let $a_n = \widetilde{P}_n(0)$ $(n = 1, 2, ...)$. In the theory of orthogonal polynomials the numbers a_n are known as reflection coefficients. It is well known that all zeros of $P_n(z)$ $(n = 1, 2, ...)$ are founded inside the unit circle (a theorem of Geronimus). Therefore, for the reflection coefficients the following is true $a_0 = 1$, $\mid a_n \mid < 1$ $(n = 1, 2, ...)$. We are interested by considering a probability measure μ for which so-called Szegö condition fails, namely $\sum_{n=1}^{\infty} \mid a_n \mid^2 = \infty$. In this case the family of the orthogonal polynomials $P_n(z)$ $(n = 0, 1, ...)$ forms an orthogonal basis in $L_2(T, \mu)$, and information about probability measure μ can be derived from the spectral properties of the multiplication operator

$$(U_\mu f)(x) = zf(z) \ (z \in T; \ f \in L_2(T, \mu)).$$

The matrix representation of $U(\mu)$ in respect to the basis $(P_n(z))$ is the following

$$U_\mu \sim [u_{jk}], u_{kj} = (U_\mu P_j, P_k) \ (j, k = 0, 1, ...),$$

where $u_{kj} = -a_{j+1} \ \bar{a}_k \ \prod_{l=k+1}^{j} (1- \mid a_k \mid^2)^{1/2}$ for $k = 0, 1, ..., j$, $u_{kj} = (1- \mid a_{j+1} \mid^2)^{1/2}$ for $k = j + 1$, and $u_{kj} = 0$ for $k = j + 2, ...$.

A matrix of this form is called a Hessenberg matrix.

We study the spectral properties of the operator H generated by a Hessenberg type matrix. The main attention is paid on the study of the structure of the spectrum of H.

Samuel multiplicity and Fredholm theory

Jörg Eschmeier

Let $T = (T_1, \ldots, T_n)$ be a commuting tuple of continuous linear operators on a complex Banach space X. If T is Fredholm, then for a sufficiently small open neighbourhood U of $0 \in \mathbb{C}^n$, the discontinuity points of the function $z \mapsto \dim H^p(z - T, X)$ form a nowhere dense analytic subset S_p of U. We show that

$$\dim H^p(z-T,X) = \lim_{k\to\infty} \dim H^p(T^k,X)/k^n$$

for $z \in U \setminus S_p$ and that these numbers are the Hilbert-Samuel multiplicities of the coherent cohomology sheaves $H^p(z-T,\mathcal{O}^X_{\mathbb{C}^n})$ at $z=0$. Thus we extend results of Xiang Fang and Douglas and Yan, either proved for $p=n$ or for commuting pairs of operators, to the general multivariable case.

Moment problem with contractive solutions

Zenon J. Jabłoński

The results presented here are contained in **[1,2]**.

Let $\mathcal{D}$ be a complex inner product spaces and let $\mathcal{H}$ be a complex Hilbert space. We denote by $\boldsymbol{L}(\mathcal{D},\mathcal{H})$ (resp. $\boldsymbol{B}(\mathcal{H})$) the set of all linear (resp. bounded linear) operators from $\mathcal{D}$ to $\mathcal{H}$ (resp. from $\mathcal{H}$ to $\mathcal{H}$). An operator moment problem entails determining whether, for a given family $\{A_{\boldsymbol{x}}\}_{\boldsymbol{x}\in\mathbb{Z}[X]_+} \subset \boldsymbol{L}(\mathcal{D},\mathcal{H})$ of operators, there exists a family $\boldsymbol{T} = \{T_\xi\}_{\xi\in X} \subset \boldsymbol{B}(\mathcal{H})$ of commuting contractions having a unitary dilation and such that

$$A_{\boldsymbol{x}} = \boldsymbol{T}^{\boldsymbol{x}} A_{\boldsymbol{0}}, \quad \boldsymbol{x} \in \mathbb{Z}[X]_+.$$

The lecture will be a survey of results concerning an operator moment problem.

1. Z. J. Jabłoński, F. H. Szafraniec, J. Stochel, Unitary propagation of operator data, *to appear in* Proc. Edinburgh Math. Soc.
2. Z. J. Jabłoński, Moment problem with contractive solutions - the regular case, *to appear in* Proc. Amer. Math. Soc.

A remark on the absolutely continuous part of the spectrum of unbounded Jacobi matrices

Jan Janas

Given two sequences of positive numbers a_n and real numbers b_n the symmetric operator J in $l^2(N)$ is defined by

$$(1)(Ju)_n = a_{n-1}u_{n-1} + b_n u_n + a_n u_{n+1}, n > 1,\ (Ju)_1 = b_1 u_1 + a_1 u_2.$$

Generalized eigenvectors of J are nontrivial solutions of the infinite system of equations

$$(2)(Ju)_n = Eu_n, E \in R, n > 1.$$

The well known subordination theory of Gilbert-Khan-Pearson describes the support of the absolutely continuous part of J as the set of all $E \in R$ whose generalized eigenvectors are not subordinated solutions of (2). Applying a general result of measure theory we shall prove that the set $E \in R$ such that any eigenvector u_n of E satisfies: $|u_n|^2 \le Ca_n^{-1}$, is contained in the absolutely continuous spectrum of J.

Compressions of Stable Contractions

László Kérchy

Joint work with Vladimir Müller

Let $\mathcal{H}$ be a complex Hilbert space, and let $\mathcal{L}(\mathcal{H})$ denote the C^*-algebra of all bounded linear operators acting on $\mathcal{H}$. An operator $T \in \mathcal{L}(\mathcal{H})$ is called *stable*, if its positive powers converge to zero in the strong operator topology. The Banach–Steinhaus Theorem shows that each stable operator T is power bounded. Let $\mathcal{P}(\mathcal{H})$ stand for the set of all orthogonal projections in $\mathcal{L}(\mathcal{H})$. We are interested in the question *whether the stability of* $T \in \mathcal{L}(\mathcal{H})$ *implies the stability of the operator* $T_P := PTP \in \mathcal{L}(\mathcal{H})$, for a projection $P \in \mathcal{P}(\mathcal{H})$. One can check with a simple example that the answer is negative in such generality.

It is natural to make the assumption that $T \in \mathcal{L}(\mathcal{H})$ is a contraction: $\|T\| \leq 1$. If T is a strict contraction then the answer is obviously positive, since $\|T_P\| \leq \|T\| < 1$ implies the uniform stability of T_P. It is also easy to verify that the stability of the contraction T is inherited by T_P if the projection P has finite rank.

By a well-known theorem of C. Foias, restrictions of the infinite dimensional backward shift B_∞ provide all stable contractions. Thus the previous question is equivalent to the problem *whether all compressions of the infinite dimensional backward shift* B_∞ *are stable.* In view of a general theorem on contractions, it can be easily justified that if a non-stable contraction T can be dilated to B_∞, then contractions similar to the simple unilateral shift S_1 can also be dilated to B_∞.

It is shown that there do exist non-stable unilateral weighted shifts, similar to S_1, which can be dilated even to the one-dimensional backward shift B_1. Actually, a complete characterization of such unilateral weighted shifts is given. Dilations of bilateral weighted shifts into backward shifts are also studied.

Rapid boundary stabilization of linear evolutionary systems

Vilmos Komornik

We prove that under rather general assumptions an exactly controllable problem is uniformly stabilizable with arbitrarily prescribed decay rates. Our approach is direct and constructive and avoids many technical difficulties associated with the usual methods based on Riccati equations. We also discuss the analogy between our result and of the Hilbert Uniqueness Method of J.-L. Lions. We give applications for the wave equation and for Petrovsky systems.

References

[1] V. Komornik, *Rapid boundary stabilization of linear distributed systems*, SIAM J. Control Optim. 35 (1997), 1591–1613.

[2] V. Komornik and P. Loreti, *Fourier Series in Control Theory*, Springer-Verlag, New York, 2005.

[3] J.-L. Lions, *Exact controllability, stabilizability, and perturbations for distributed systems*, SIAM Rev. 30 (1988), 1–68.

Almost stability of unbounded semigroups of operators

Zoltán Léka

In this talk, we present a method that makes possible to associate an isometric representation to unbounded representations of abelian semigroups, satisfying certain regularity condition. As an application, we prove an Arendt–Batty–Lyubich–Vũ-type stability theorem using the limit isometric semigroup. We obtain that the weighted orbits converge to zero in terms of almost convergence. The peripheral spectrum is introduced by means of a limit functional associated with the representation.

A lifting commutant theorem for unbounded quasinormal operators

Witold Majdak

Significant progress in the study of unbounded subnormal operators and their minimal normal extensions of different types has been achieved due to the trilogy [2], [3], [4]. More recent article [1] supplies numerous lifting strong (or symmetric) commutant theorems for unbounded subnormal operators. This presentation is the continuation of investigations started in [1]. We focus our attention on the class of quasinormal operators. Our main purpose is to obtain the following unbounded version of Yoshino's criterion (cf. [5, Theorem 4]) on lifting strong commutant of a quasinormal operator:

THEOREM. *Let $\mathcal{H}$ and $\mathcal{K}$ be Hilbert spaces such that $\mathcal{H}$ is isometrically included in $\mathcal{K}$. Take a quasinormal operator Q (acting in $\mathcal{H}$) and its minimal normal extension N of spectral type (acting in $\mathcal{K}$). Let $Q = V|Q|$ be the polar decomposition of Q. If $T \in \boldsymbol{B}(\mathcal{H})$, then the following conditions are equivalent:*

(i) *there exists a unique operator $\widehat{T} \in \boldsymbol{B}(\mathcal{K})$ such that $T \subseteq \widehat{T}$ and $\widehat{T}N \subseteq N\widehat{T}$,*

(ii) *$T|Q| \subseteq |Q|T$ and $TV = VT$.*

Moreover, if (i) *holds, then $TQ \subseteq QT$ and $\|T\| = \|\widehat{T}\|$.*

We also discuss relationships between minimal normal extensions of spectral and cyclic type of an unbounded quasinormal operator and we establish some properties (as for example tightness) of such extensions.

References

[1] J. Stochel, Lifting strong commutants of unbounded subnormal operators, *Integral Equations and Operator Theory* **43** (2002), 189-214.

[2] J. Stochel, F. H. Szafraniec, On normal extensions of unbounded operators. I, *J. Operator Theory* **14** (1985), 31-55.

[3] J. Stochel, F. H. Szafraniec, On normal extensions of unbounded operators. II, *Acta Sci. Math. (Szeged)*, **53** (1989), 153-177.

[4] J. Stochel, F. H. Szafraniec, On normal extensions of unbounded operators. III: Spectral properties, *Publ. Res. Inst. Math. Sci.* **25** (1989), 105-139.

[5] T. Yoshino, On the commuting extensions of nearly normal operators, *Tohôku Math. J.* **25** (1973), 263-272.

Nonlinear problems with odd number of bifurcating solutions

Ľubomír Marko

We deal with the nonlinear bifurcation problem $u - \lambda Lu + N(u) = 0$ in Hilbert space, where λ is the real parameter, $L : H \to H$ is linear, positive, self-adjoint, compact operator,

$N : H \to H$ is bounded, nonlinear, compact operator of odd degree. The operators L, N fulfil some additional assumptions. We suppose $\lambda_1 < \lambda_2 < \cdots < \lambda_n < \dots$, where $\lim_{n\to\infty} \lambda_n = \infty$, be the set of simple eigenvalues of linearized problem $u - \lambda Lu = 0$ in H. We prove the local and the global existence of nontrivial solutions bifurcating from the trivial solution in the point $(0, \lambda_i)$. We present also exact solutions multiplicity result in dependence on the parameter λ.

New results on linear presers problems

Mostafa Mbekhta

Let H be an infinite-dimensional complex separable Hilbert space and $\mathcal{B}(H)$ the algebra of all bounded linear operators on H. In this talk, we discuss the following new results:

I) Let $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ be a bijective continuous unital linear map preserving generalized invertibility in both directions. Then the ideal of all compact operators is invariant under ϕ and the induced linear map on the Calkin algebra is either an automorphism or an antiautomorphism.

II) Let $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ be a surjective linear map. Then ϕ preserves the essential spectrum if and only if the ideal of all compact operators is invariant under ϕ and the induced linear map φ on the Calkin algebra is either an automorphism, or an anti-automorphism.

Moreover, we prove that either

$$ind(\phi(T)) = ind(T) \quad \text{or} \quad ind(\phi(T)) = -ind(T),$$

for every Fredholm operator T .

III) If $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ is a surjective unital linear map on $\mathcal{B}(H)$, then $m(\phi(T)) = m(T)$ for every $T \in \mathcal{B}(H)$ if and only if $q(\phi(T)) = q(T)$ for every $T \in \mathcal{B}(H)$ if and only if there exists an unitary operator $U \in \mathcal{B}(H)$ such that $\phi(T) = UTU^*$ for all $T \in \mathcal{B}(H)$.

Here $m(T)$ and $q(T)$ denote respectively the minimum modulus and the surjectivity modulus for every $T \in \mathcal{B}(H)$.

Additivity of multiplicative maps on structures of linear operators

Lajos Molnár

The additivity of multiplicative or multiplicative-like transformations on rings and algebras has been studied by several authors. In this talk we deal mainly with the additivity of bijective maps on algebraic structures of linear operators which are multiplicative with respect to the so-called Jordan triple product. This operation plays important role in ring theory and in the mathematical descriptions of quantum mechanics. Among others we present the following result.

Theorem. *Let $\mathcal{A}, \mathcal{B}$ be von Neumann algebras. Suppose that $\mathcal{A}$ does not have a commutative direct summand. Denote $\mathcal{A}_s$ and $\mathcal{B}_s$ the self-adjoint parts of $\mathcal{A}$ and $\mathcal{B}$, respectively. Let $\phi : \mathcal{A}_s \to \mathcal{B}_s$ be a bijective map which satisfies*

$$\phi(ABA) = \phi(A)\phi(B)\phi(A) \qquad (A, B \in \mathcal{A}_s).$$

Then we have direct decompositions

$$\mathcal{A} = \mathcal{A}_1 \oplus \mathcal{A}_2 \oplus \mathcal{A}_3 \oplus \mathcal{A}_4 \quad \text{and} \quad \mathcal{B} = \mathcal{B}_1 \oplus \mathcal{B}_2 \oplus \mathcal{B}_3 \oplus \mathcal{B}_4$$

within the category of von Neumann algebras and bijective linear maps

$$\Phi_1 : \mathcal{A}_1 \to \mathcal{B}_1,\ \ \Phi_2 : \mathcal{A}_2 \to \mathcal{B}_2,\ \ \Phi_3 : \mathcal{A}_3 \to \mathcal{B}_3,\ \ \Phi_4 : \mathcal{A}_4 \to \mathcal{B}_4$$

*such that Φ_1, Φ_2 are linear *-algebra isomorphisms, Φ_2, Φ_4 are linear *-algebra antiisomorphisms and*

$$\phi = \Phi_1 \oplus (-\Phi_2) \oplus \Phi_3 \oplus (-\Phi_4)$$

holds on $\mathcal{A}_s$.

New classes of Jacobi matrices with absolutely continuous spectrum

Wojciech Motyka

The talk will concern the question of absolutely continuous Jacobi matrices. Two different approaches to this question will be presented. The first approach uses a new result on asymptotic behavior of generalized eigenvectors. The second one employs ideas of recent paper by R.Szwarc.

This talk is based on a joint work with J.Janas.

On hypercyclic semigroups

Vladimir Müller

Let $\mathcal{T} = T(t)_{t\geq 0}$ be a strongly continuous semigroup of operators on a Banach space X. A vector $x \in X$ is called hypercyclic for $\mathcal{T}$ if the orbit $\{T(t)x : t \geq 0\}$ is dense. We show that then x is hypercyclic for each operator $T(t_0)$, i.e., the set $\{T(nt_0)x : n = 0, 1, \dots\}$ is dense.

Subdecomposability and the Kato-type spectrum

Michael M. Neumann

This talk centers around a classical property that was introduced by Errett Bishop about fifty years ago and only recently gained a central role in the local spectral theory of bounded linear operators on Banach spaces. In fact, Bishop's property (β) characterizes the restrictions of decomposable operators to closed invariant subspaces and thus may be viewed as the proper extension of the notion of subnormality from Hilbert to Banach spaces. Here we explore the extent to which Bishop's property (β) holds on certain parts of the spectrum of an operator. As one of the applications, we obtain a remarkably sharp solution to the problem of the equality of the essential spectra for a pair of quasi-similar operators on Banach spaces. Our approach is elementary in the sense that the use of sheaf theory is avoided completely. Our main tools are provided by the theory of semi-regular and Kato-type operators.

Uniqueness of some joint spectra

Krzysztof Rudol

It often happens that certain "canonically defined" parts of the spectrum, when considered within a special class of operators -actually fill in the entire set. The advantage is obvious whenever the description of such a part is simpler, since often only one of the possible causes of non-invertibility is then involved. In the case of joint spectra the gain becomes even more essential, especially when there is no practical way of finding the "whole joint spectrum".

In my talk I will be tracing some positive examples related to commuting pure isometries, where different parts of joint spectra are equal. In particular, the closed defect spectrum and Taylor's joint spectrum are compared to the joint spectrum with respect to the inverse-closed Banach algebra generated by a given pair of isometries.

In case of commuting pure subnormal operators a negative example, showing that the expected uniqueness actually fails is presented.

Lebesque Decomposition for forms

Zoltán Sebestyén

The Lebesque decomposition stands well known for measures: a measure ν decomposes into unique absolutely continuous and singular parts with respect to another measure μ as follows.

$$\nu = \nu_a + \nu_b$$

Let the forms m, n, n_a and n_b be determined as the inner products of the corresponding L^2 - spaces

$$L^2(\mu),\ L^2(\nu),\ L^2(\nu_a),\ L^2(\nu_b)$$

restricted to the vector space of simple functions, the linear hull of characteristic functions of measurable sets of finite measures.

According to B. Simon (1978) the form n decomposes with respect to the form m as follows

$$n = n_r + n_s,$$

where n_r and n_s are the closable (regular) and singular parts , respectively, of the form n. Here n_r is the unique closable form with respect to m which is bounded by n.

Our problem is whether $n_a = n_r$, equivalently $n_b = n_s$.

Striving for normality: local spectral radii and C_0-semigroups

Jan Stochel

New criteria for essential normality of unbounded Hilbert space operators are supplied in a twofold manner: either in terms of local spectral radius or via C_0-semigroups. Classes of operators of certain types related to local spectral radius are discussed together with illustrative examples.

Talk is based on a paper by D. Cichoń, Il Bong Jung and Jan Stochel (with the same title).

Subnormality of unbounded operators: never enough of it

Franciszek H. Szafraniec

A few words about an attempt of fitting in the well know Ando's construction to unbounded circumstances.

Maps on idempotents

Peter Šemrl

There are two natural relations on the set of all bounded idempotent linear operators on a Banach space: partial order and orthogonality. We study maps on idempotents preserving one of these two relations. The structural results for such maps have many applications in the theory of linear preservers, automorphisms and local automorphisms of operator algebras, physics, and geometry.

Orbit-reflexive Operators

Jan Vršovský

Joint work with Vladimír Müller

Let T be a bounded linear operator on a (real, complex) Banach space X. Recall that T is reflexive if every bounded linear operator A belongs to the closure of $\mathrm{Span}\{T^n\colon n \in \mathbb{N}\}$ in the strong operator topology whenever $Au \in \overline{\mathrm{Span}\{T^n u\colon n \in \mathbb{N}\}}$ for each $u \in X$. Similarly, T is orbit-reflexive if every bounded linear operator A belongs to the closure of $\{T^n\colon n \in \mathbb{N}\}$ in the strong operator topology whenever $Au \in \overline{\{T^n u\colon n \in \mathbb{N}\}}$ for each $u \in X$. While the notion of reflexivity is connected to the problem of invariant subspaces, orbit-reflexivity is in the same way connected to the problem of invariant subsets.

We show two conditions concerning the orbit-reflexivity property. Firstly, if the powers of the operator grow sufficiently quickly, namely if

$$\sum_{n=1}^{\infty} \frac{1}{\|T^n\|} < \infty \qquad \left(\text{or } \sum_{n=1}^{\infty} \frac{1}{\|T^n\|^2} < \infty \text{ in a complex Hilbert space}\right),$$

then there is a dense set of points x with $\|T^n x\| \to \infty$ as $n \to \infty$, and T is orbit-reflexive. Secondly, if there is a point $x \in X$ such that the closure of its orbit $\overline{\{T^n x\colon n \in \mathbb{N}\}}$ is countable then either T has a nontrivial closed hyperinvariant subspace or T is orbit-reflexive.

Domination and commutators of block operator matrices

Michał Wojtylak

The idea of domination of operators in Hilbert spaces is due to Nelson [2]. The present talk is mainly based on the paper [4]. We continue the research from [1], by investigating commutators of block operator matrices. Our main result is the following.

Theorem. *Assume that S (A) is a selfadjoint (resp. a symmetric) operator in the orthogonal sum of two Hilbert spaces $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1$. Moreover, let $K \in \mathbf{B}(\mathcal{H})$ be such that $K(\mathcal{H}_0) \subseteq \mathcal{H}_0$ and let $U \subseteq \rho(S+K)$ be unbounded and such that $|z|\|(S + B - z)^{-1}\| \leq \mathrm{const}$ for $z \in U$. If for some $m \in \mathbb{N}$ $\mathcal{H}_0 \subseteq \mathcal{D}(S^m) \subseteq \mathcal{D}(A)$, $\mathcal{E}$ is a core for S^m and*

$$\sup_{z \in U} |z| \| [(S + K - z)^{-1}, \overline{A\,|_{\mathcal{E}}}] \| < +\infty \tag{1}$$

then A is essentially selfadjoint on $\mathcal{E}$.

We also consider the special case when the operators S and A commute in the way suggested in [3]. This leads to extension of the results from [1] and [3] onto Krein spaces.

References

[1] D. Cichoń, J. Stochel, F.H. Szafraniec, Noncommuting domination, *Oper. Theory Adv. Appl.* 154 (2004), 19-33 .
[2] E. Nelson, Analytic vectors, *Ann. of Math.,* 70 (1959), 572-615.
[3] J. Stochel, F. H. Szafraniec, Domination of unbounded operators and commutativity, *J. Math. Soc. Japan,* 55 No.2, (2003), 405-437.
[4] M. Wojtylak, Noncommuting domination in Krein spaces via commutators of block operator matrices, (to appear).

Reflexivity of the space of locally intertwining operators

Michal Zajac

Let H, H' be complex separable Hilbert spaces. Let $B(H, H')$ ($B(H)$, if $H = H'$) denote the space of all bounded linear operators $T : H \to H'$.

Recently, Janko Bračič has studied the reflexivity of local commutants of a given operator $T \in B(H)$ at a given vector $\mathbf{e} \in H$, $C(T, \mathbf{e}) = \{A \in B(H) : AT\mathbf{e} = TA\mathbf{e}\}$.

It is known (M. Zajac, Proceedings of the 2nd Workshop on Functional Analysis, Nemecká, 1999) that for finite-dimensional H, H' the space

$$I(T, T') = \{A \in B(H, H') : AT = T'A\}$$

is reflexive if and only if the greatest common divisor of the minimal polynomials of T and T' has no zeroes of multiplicity more than 1.

We are going to present local version of this result, i.e. we shall give conditions under which the set of locally intertwining operators at a vector $\mathbf{e} \in H$

$$I(T, T'; \mathbf{e}) = \{A \in B(H, H') : AT\mathbf{e} = T'A\mathbf{e}\}$$

is reflexive.

It is known that quasi-similarity preserves reflexivity of $I(T, T')$. An example showing that this is not true for hyper-reflexivity will be given. More precisely, we shall show an example of pairs of operators (T, T'), (S, S') (in infinite-dimensional spaces) satisfying conditions

(1) T, T' are quasisimilar to S, S', respectively.
(2) $I(T, T')$ is not hyper-reflexive.
(3) $I(S, S')$ is hyper-reflexive.

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Eigenvalue distribution for some classes of Jacobi matrices

Lech Zielinski

Let $(a(n))_{n\in\mathbf{Z}}$, $(b(n))_{n\in\mathbf{Z}}$ be real valued sequences and assume that P is a self-adjoint operator in the Hilbert space $l^2(\mathbf{Z})$ acting according to the formula

$$(Px)(n) = b(n)x(n-1) + b(n+1)x(n+1) + a(n)x(n)$$

for $(x(n))_{n\in\mathbf{Z}}$ belonging the domain of P.

We are interested in the case when the sequence $(a(n))_{n\in\mathbf{Z}}$ has a polynomial growth, i.e. we assume that it is possible to find constants $C \geq c > 0$ such that the estimate

$$c|n|^c - C \leq a(n) \leq C(1 + |n|)^C$$

holds for all $n \in \mathbf{Z}$. We assume moreover that $b(n)/a(n) \to 0$ when $|n| \to \infty$, which ensures the fact that the spectrum of P is discrete and bounded from below.

For $\lambda \in \mathbf{R}$ let $N(P, \lambda)$ denote the counting function defined as the number of eigenvalues smaller than λ (counted with multiplicities).

The aim of this talk is to discuss different regularity assumptions on $(a(n))_{n\in\mathbf{Z}}$, $(b(n))_{n\in\mathbf{Z}}$ that allow to compare the behaviour of $N(P, \lambda)$ and

$$N_0(\lambda) = \sum_{n\in\mathbf{Z}} \int_{\{\xi\in[0;\, 2\pi]:\, a(n)+2b(n)\cos\xi<\lambda\}} \frac{d\xi}{2\pi}.$$

On operator matrix methods in the theory of operator algebras

László Zsidó

A quasitrace on a C^*-algebra A is a function $\tau : A \longrightarrow \mathbb{C}$ such that

$\tau(x^*x) = \tau(x\,x^*) \geq 0\,, \qquad x \in A\,,$
$\tau(a + ib) = \tau(a) + i\,\tau(b)$ for all self-adjoint $a\,, b \in A\,,$
τ is linear on any commutative $*$-subalgebra of $A\,.$

A quasitrace τ on A is called 2-quasitrace if there exists a quasitrace τ_2 on the 2×2 matrices over A such that

$$\tau(x) = \tau_2\left(\begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix} \right), \qquad x \in A\,.$$

It is known that not every quasitrace is 2-quasitrace (E. Kirchberg), but it is still an open question (I. Kaplansky) whether every 2-quasitrace on a C^*-algebra A is always linear (U. Haagerup proved this for exact A).

It is of interest to prove for 2-quasitraces on C^*-algebras even weaker properties than the linearity, which would be implied by the linearity. The extendibility of a 2-quasitrace τ on A to the 2×2 matrices over A allows to use operator matrix relations in proving relations between the values of τ in certain elements of A. The purpose of my talk is to discuss such operator matrix methods.