Ergodic domination in ordered Banach algebras with disjunctive products

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An operator T is said to be *ergodic* if the sequence $\frac{1}{n}\sum_{k=0}^{n-1} T^k$ is uniformly convergent. In [2] Caselles showed that if S and T are positive operators on a Banach lattice such that $0 \leq S \leq T$, T is ergodic and the spectral radius of T is a Riesz point of the spectrum of T, then S is ergodic. In [3] a version of this result, which holds in general ordered Banach algebras (OBAs) was presented. A key assumption in this theorem was, however, that the spectral radius in an associated quotient algebra is weakly monotone, a condition which is naturally satisfied in the ordered Banach algebra of regular operators on a Dedekind complete Banach lattice E, but generally fails for the bounded linear operators on E.

This phenomenon prompted the open question of whether such an ergodic domination theorem can be proven without a weak monotonicity assumption in the context of general OBAs (i.e. using only Banach algebra techniques) — see [4] and [5]. We address this issue by utilising Alekhno's groundbreaking work on irreducibility, Frobenius normal forms and disjunctive products in OBAs — see [1]. In particular, we prove that if $0 \leq a \leq b$ in an OBA, then under natural assumptions ergodicity of b implies that there exists a spectral block of a that is ergodic. This result involves no operator theoretic techniques; neither does it rely on any weak monotonicity assumptions. It partially resolves the mentioned open question and applies, among other things, to the algebra of bounded linear operators on a Banach lattice with order continuous norm.

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[5] A. W. Wickstead: Ordered Banach algebras and multi-norms: some open problems, *Positivity*, 21 (2017), 817–823.