Small Workshop on Operator Theory

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The Workshop is organized by:
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Abstracts

Spectral properties of averaging operators

E. Albrecht

In this report on joint work with V. Miller (Mississippi State) we investigate the global and local spectral properties of various averaging operators on Banach spaces of analytic functions. In particular we obtain a more direct proof for the result of Miller, Miller and Smith that the classical Cesàro operator has Bispop's property (β) on H^p.

Decomposition of pairs of commuting quasi-normal partial isometries

Z. Burdak

The intersection of class class quasinormal operators and class of power partial isometries is considered. It turns out to be also subclass of power partial isometries. The decomposition theorem for pair of commuting quasinormal partial isometries will be presented. Some examples will be shown.

Spectral properties of the Pauli operator

P. A. Cojuhari

The Pauli wave equations for a particle in an electromagnetic field is typical written in the physics literature [1] (see also [2]) as

$$i\hbar\frac{d\Psi}{dt} = P_0\Psi + Q(x)\Psi$$

in which

$$P_0\Psi_k = -\frac{\hbar^2}{2m}\triangle\Psi_k - \frac{i\hbar e}{mc} < A, \nabla\Psi_k > + \frac{e^2}{2mc^2}|\vec{A}|^2\Psi_k(k=1,2),$$

$$Q(x) = \mu_0 \begin{pmatrix} H_3(x) - \frac{e}{\mu_0}q(x) & H_1(x) + iH_2(x) \\ H_1(x) - iH_2(x) & -H_3(x) - \frac{e}{\mu_o}q(x) \end{pmatrix}$$

where $\Psi = (\Psi_1, \Psi_2)$ is a two-component vector function of $x \in R^3$, $\vec{A} = (A_1, A_2, A_3)$ is the given vector potential corresponded to the magnetic field, $|\vec{A}|^2 = \sum_{j=1}^{3}|A_j|^2$, $H = (H_1, H_2, H_3)$ is the magnetic field strength, $q(x)$, $x \in R^3$, is a scalar potential (potential energy). In addition, the following conditions $H = rot\vec{A}$ and $div\vec{A} = 0$ for the functions $\vec{A}$ and $\vec{H}$ are always assumed.

The operator P defined in the space $L_2(R^3, C^2)$ by

$$P\Psi = P_0\Psi + Q(x)\Psi, \Psi \in Dom(P)(= W_2^2(R^3))$$

is known as the Pauli operator corresponding to the above Pauli system.

Our purpose is to study the spectral properties of the Pauli operator P. The main attention is paid on the study of the structure of its spectrum. In particular, some criteria for the finiteness of the discrete spectrum $\sigma_d(P)$ are formulated. The main results are obtained within the framework of the methods developed by us in [3].

References

[1] V. A. Foc, *Introduction to the Quantum Mechanics,* LGU, 1956.
[2] I. M. Glazman, *Direct Methods of Qualitative Spectral Analysis of Singular Differential Operators,* M. Puz-Mat.
[3] P. A. Cojuhari, *On the finiteness of the discrete spectrum of some matrix pseudodifferential operators,* Izv. Vysh. Ucebn. Zaved. Mat. 1, 1983, p. 42-50.

Characteristic functions for row contractions

Jörg Eschmeier

A theorem of Sz.-Nagy and Foias shows that the characteristic function $\theta_T(z) = -T + zD_{T^*}(1 - zT^*)^{-1}D_T$ of a completely non-unitary contraction T is a complete unitary invariant for T. In the planned lecture we extend this result to the case of a pure commuting contractive tuple using a natural generalization of the characteristic function to an operator-valued analytic function on the open unit ball in $\mathbb{C}^n$. This function is related to the curvature invariant introduced by Arveson.

Sharp estimates of matrix elements of the resolvent of general Jacobi matrices

Jan Janas

This talk is devoted to the spectral theory of unbounded Jacobi matrices and consists of two complementary parts:

In the first part we consider general unbounded self-adjoint Jacobi matrices J on l^2 and assume that λ lies in a spectral gap of J. We will use a "discrete" and rather simple version of a technique introduced by Combes and Thomas to prove upper bounds on the exponential decay of generalized eigenfunctions of J to λ. The decay bound for eigenfunctions of Schrödinger operators found by them improved on longstanding bounds. Thus, what we are doing here, is to apply the improved "Combes-Thomas-Hislop method" to unbounded Jacobi matrices.

While the result is quite general and its proof, due to the discrete one-dimensional setting, quite elementary, the obtained bounds are remarkably sharp in several respects. This will be understood in the second part of the talk, where we will consider a concrete class of unbounded Jacobi matrices for which the exact asymptotics of generalized eigenfunctions can be obtained.

Quasinormality and reflexivity

K. Kliś

This talk will be devoted to reflexivity of weak* closed subspaces generated by powers of a quasinormal operator. It will be also shown that quasinormal operators are hyperreflexive.

Liftings of intertwining relations for subnormal operators

Witold Majdak

Co-authors: **Zoltán Sebestyén** (Eötvös University, Budapest)
Jan Stochel (UJ, Kraków)

We propose new characterizations of operators which lift to operators intertwining minimal normal extensions of given subnormal operators. Next, we study intertwining relations of local nature, where ,,local" refers to cyclic parts of subnormal operators under the consideration. The main goal of the talk is to find the relationship between the existence of liftings of local and global intertwining relations for subnormal operators. Some examples illustrating the subject of the talk are also presented.

A characterization of subscalar operators

Vladimir Müller

Invertible extensions of operators and invertibility of Banach algebras elements in their extensions will be disscused. The results will be applied to characterizations of subscalar and subdecomposable operators.

Power partial isometries, consistant operators and reflexivity

M. Ptak

Necessary and sufficient conditions for reflexivity of power partial isometry are given. One of the condition is expressed by rank two operators. The new class of operators, consistent operators, will be helpful. The reflexivity of such operators will be studied.

Joint work with EDWARD A. AZOFF, WING SUET LI, MOSTAFA MBEKHTA

Backward extensions of subnormal operators

Jan Stochel

The concept of backward extension for subnormal weighted shifts was introduced by R. Cutro in 1990. It was intensively studied by T. Hoover, I. Jung and A. Lambert. The goal of my talk is to show how this concept can be generalized to arbitrary subnormal operators including cyclic ones. Several differences and similarities in these contexts are explored, with emphasis on the structure of the underlying measures (the latter in the case of cyclic subnormal operators).

References

[1] I. B. Jung, A. Lambert, J. Stochel, *Backward extensions of subnormal operators,* Proc. Amer. Math. Soc. 132 (2004), 2291-2302.

Subnormality and cyclicity

F. H. Szafraniec

Relations between subnormality of an operator and that of its cyclic ingredients will be discussed.

Stability of the exponential equation in commutative Banach algebras

Jacek Tabor

The problem of stability of additivity was posed by S. Ulam [4]:
Let G be a group and X be a Banach space. Does for every ϵ there exist δ such that for every function $f : G \to X$ satisfying

$$\|f(x+y) - f(x) - f(y)\| \leq \delta \quad for\ x, y \in G$$

there exist an additive function $a : G \to X$ such that

$$\|f(x) - a(x)\| \leq \epsilon \quad for\ x \in G?$$

The positive answer was given by Hyers [2] and led to the investigation of stability of functional equations.

In this talk we deal with the stability of exponential equation

$$f(x+y) = f(x)f(y) \quad \text{for } x, y \in G.$$

The result we discuss is a generalization of [1] and, roughly speaking, goes as follows:

Theorem. Let G be an abelian group, let A be a commutative Banach algebra and let $\epsilon \in [0, 1/2)$. Let $f : G \to A$ be such that

$$\|\frac{f(x)f(y)}{f(xy)} - 1\| \leq \epsilon \quad \text{for } x, y \in G.$$

Then there exist a multiplicative function $m : g \to A$ such that

$$\|\frac{f(x)}{m(x)} - 1\| \leq \frac{\epsilon}{1-\epsilon} \quad \text{for } x \in G.$$

References

[1] R. Ger, P. Šemrl, *The stability of the exponential equation,* Proc. Amer. Math. Soc. **124** (1996), 779-787.

[2] D. H. Hyers, *On the stability of the linear functional equation,* Proc. Natl. Acad. Sci. USA **16** (1941), 222-224.

[3] Jacek Tabor, Hyers Theorem and the Cocycle Property, in *Functional equations - Results and Advances.* Advances in Mathematics, Kluwer Academic Publishers, Boston/Dordrecht/London, 2002, 275–291.

[4] S. Ulam, *A collection of Mathematical Problems,* Interscience Publ., New York 1960.

Reflexivity and hyperreflexivity of the space of operators intertwining a pair of contractions

Michal Zajac

First we review known results on the reflexivity of the commutant $\{T\}'$ of a C_0-contraction T and of the space $I(T, T')$ of all operators intertwining a pair T, T' of c.n.u. weak contractions.

Next some results and open problems on the hyperreflexivity of above mentioned spaces will be given. In particular, if $T = \bigoplus_{n=1}^{\infty} T_n$ and $T' = \bigoplus_{n=1}^{\infty} T'_n$ we study relations between the hyperreflexivity of of $I(T, T')$ and of $I(T_i, T'_j)$.

Extremal properties of the Volterra operator

Jaroslav Zemanek

We intend to show that the Volterra operator, like its inverse - the derivative, turns out to be extremal in various aspects of operator theory. This will be demonstrated by the behaviour of powers, their Cesaro means, and analytic properties of the resolvents. The talk is based on recent joint works with Alfonso Montes-Rodriguez, Juan Sanchez Alvarez-Dardet, Yuri Tomilov, and Dashdondog Tsedenbayar.