4^{th} Small Workshop on Operator Theory

8–12 July, 2014
Kraków, Poland

The Workshop is organized by:

Department of Applied Mathematics
University of Agriculture in Krakow

Scientific Committee (Local):

Petru A. Cojuhari (AGH University of Science and Technology, Krakow)

Jan Janas (Polish Academy of Sciences, Krakow)

Marek Ptak (University of Agriculture in Krakow)

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Piotr Dymek

Kamila Kliś-Garlicka

Marta Majcherczyk

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Artur Płaneta

The conference will take place in the Main Building of the University of Agriculture in Krakow (al. Mickiewicza 24/28). Registration office will be open on Tuesday from 8.00 till 15.00 in the same building on the first floor. The opening of the conference will start at 9.00 in the room 120. Plenary talks will be held in the room 120, parallel sessions in rooms 120, 245.

An excursion to Wieliczka and Niepolomice is planned for Thursday (10$^{\text{th}}$ July). A bus will leave at 13.45 from the parking in front of the Conference building. The conference dinner will be held at Bona Restaurant in the Royal Castle in Niepolomice just after the excursion (around 18.15). We plan to return to Krakow around 22.30.

There is a wireless connection available. To get the access choose network called *WISIG* and enter the first password. Then, you have to open any web page. You will be asked to enter another login and password. The login is *guest* and a password is the second one. Additionally, there are computers at 4$^{\text{th}}$ and 5$^{\text{th}}$ floor.

Tuesday, July 8th, morning

Conference Hall: al. Mickiewicza 24/28

8:00 – 8:55	Registration
9:00 – 9:15	Opening
	Plenary session (Room 120) Chair: David Larson
9:15 – 10:05	**Raúl E. Curto** *Berger measures for transformations of subnormal weighted shifts*
10:15 – 11:05	**Cristina Câmara** *Kernels of Toeplitz operators, maximal functions and model spaces*
11:05 – 11:35	**Coffee break (Room 127)**
11:35 – 12:25	**László Kérchy** *Functional commutant of asymptotically cyclic contractions*
12:25 – 14:30	**Lunch**

Tuesday, July 8th, afternoon

Conference Hall: al. Mickiewicza 24/28

Parallel sessions		
	Session A (Room 120) Chair: Vladimir Müller	**Session B (Room 245)** Chair: Rangwei Yang
14:30 – 14:50	**Janusz Brzdęk** *Ulam's type stability for linear equations of higher orders*	**Rostyslav Hryniv** *Inverse spectral problems for energy dependent Sturm-Liouville equations*
14:55 – 15:15	**Anna Bahyrycz** *Application of the fixed point approach to hyperstability of some functional equation*	**Bruce Watson** *Indefinite Sturm-Liouville problems for the p-Laplacian*
15:25 – 15:45	**Maria Nowak** *De Branges-Rovnyak spaces and generalized Dirichlet spaces*	**Patryk Pagacz** *An asymmetric Putnam-Fuglede theorem for paranormal and ∗-paranormal operators*
15:50 – 16:10	**Renata Rososzczuk** *Weighted Sub-Bergman Hilbert Spaces*	**Lech Zielinski** *Approximation of large eigenvalues of unbounded Jacobi matrices by eigenvalues of finite submatrices*
16:10 – 16:40	**Coffee break (Room 127)**	
	Session A (Room 120) Chair: Yuri Tomilov	**Session B (Room 245)** Chair: Maria Nowak
16:40 – 17:00	**M. Teresa Malheiro** *Kernels of Toeplitz operators with symbols in Q-classes of matrix functions*	**Magdalena Piszczek** *Stability and hyperstability of some equations*
17:05 – 17:25	**Kamila Kliś-Garlicka** *C-symmetric operators and reflexivity*	**Jolanta Olko** *Application of the fixed point approach to stability of some functional equation*
17:35 – 17:55	**Paweł Wójcik** *On a restriction of an operator to an invariant subspace*	**Renata Malejki** *On the Baskakov-Durrmeyer type operators*

Wednesday, July 9th, morning

Conference Hall: al. Mickiewicza 24/28

	Plenary session (Room 120) Chair: Cristina Câmara
9:00 – 9:50	**David Larson** *Measures, dilations and frames*
10:00 – 10:50	**Yuri Tomilov** *On decay of operator semigroups*
10:50 – 11:20	**Coffee break (Room 127)**
11:20 – 11:50	**Petru Cojuhari** *On spectral analysis of block Jacobi matrices*
12:00 – 12:30	**Roman Drnovšek** *On semigroups of nonnegative functions and positive operators*
12:30 – 14:30	**Lunch**

Wednesday, July 9th, afternoon

Conference Hall: al. Mickiewicza 24/28

Parallel sessions		
	Session A (Room 120) Chair: Roman Drnovšek	**Session B (Room 245)** Chair: Janusz Brzdęk
14:30 – 14:50	**Zenon Jabłoński** *Unbounded Composition Operators in L^2-Spaces*	**Jacek Chmieliński** *Approximate inner product spaces*
14:55 – 15:15	**Raffael Hagger** *Numerical Ranges and Random Operators*	**Thomas Tonev** *Tween mappings between multiplicative subsets of function algebras and their spectral properties*
15:25 – 15:45	**Marek Kosiek** *Szego type decompositions of isometries*	**Ewelina Zalot** *A spectral representation for a class of non-self-adjoint banded Jacobi matrices*
15:50 – 16:10	**Zbigniew Leśniak** *On generating solutions of a generalized Volterra integral equation*	**Ewa Szlachtowska** *On the solvability of Dirichlet problem for the weighted p-Laplacian*
16:10 – 16:40	**Coffee break (Room 127)**	
	Session A (Room 120) Chair: Bruce Watson	**Session B (Room 245)** Chair: Andrzej Sołtysiak
16:40 - 17:00	**Jerzy B. Stochel** *Relationship between the holes of the supports of representing measures of Stieltjes moment sequences and their roots*	**Sasmita Patnaik** *Cartan Subalgebras of Operator Ideals*
17:05 – 17:25	**Witold Majdak** *Weighted shifts acting on directed semi-trees*	**Maria Malejki** *Asymptotics of the discrete spectrum for complex Jacobi matrices*
17:30 – 17:50	**Paweł Pietrzycki** *The equality $C^{*2}C^2 = (C^*C)^2$ is not sufficient for quasinormality of a composition operator C in L^2-space*	

Thursday, July 10th

Conference Hall: al. Mickiewicza 24/28

	Plenary session (Room 120) Chair: László Kérchy
9:00 – 9:50	**William Ross** *A partial order on partial isometries*
10:00 – 10:50	**Isabelle Chalendar** *Boudedness of some operators on spaces of holomorphic functions*
10:50 – 11:20	**Coffee break (Room 127)**
11:20 – 12:10	**Vladimir Müller** *Alternating projections on Hilbert spaces*
12:10 – 13:45	**Lunch**
13:45 – 22:00	**Excursion and Conference Dinner**

Friday, July 11th, morning

Conference Hall: al. Mickiewicza 24/28

	Plenary session (Room 120) Chair: Raúl E. Curto	
9:00 – 9:50	**Jan Stochel** *Subnormality of unbounded composition operators in* L^2 *spaces*	
10:00 – 10:50	**Rongwei Yang** *Operator theory in the Hardy space over the bidisk*	
10:50 – 11:20	**Coffee break (Room 127)**	
Parallel sessions		
	Session A (Room 120) Chair: Petru Cojuhari	**Session B (Room 245)** Chair: William Ross
11:20 – 11:50	**Maria Joita** *Crossed products by Hilbert pro-*C^**-bimodules*	**Piotr Grabowski** *The LQ-problem for infinite-dimensional systems with bounded operators*
12:00 – 12:30	**Uğur Gül** *On the* C^**-algebra generated by Toeplitz Operators and Fourier Multipliers on the Hardy space of a locally compact*	**Krzysztof Rudol** *Analytic measures and weak-star closures in general function algebras*
12:30 – 14:30	**Lunch**	

Friday, July 11th, afternoon

Conference Hall: al. Mickiewicza 24/28

Parallel sessions		
	Session A (Room 120) Chair: Jan Stochel	**Session B (Room 245)** Chair: Uğur Gül
14:30 – 14:50	**Cristina Diogo** *Removing zero from the numerical range*	**Sergii Kuzhel** *On S-matrix of Schrödinger Operators with Non-Symmetric Zero-Range Potentials*
14:55 – 15:15	**Agnieszka Kowalska** *Generalized Markov properties*	**Tomasz Beberok** *The Bergman kernel function for some Reinhardt domains*
15:25 – 15:45	**Zbigniew Burdak** *On the decomposition and the model for commuting isometries*	**Tomasz Kobos** *Minimal projections of* 3*-dimensional normed spaces*
15:50 – 16:10	**Attila Szalai** *Spectral behaviour of quasianalytic contractions*	**Piotr Niemiec** *Functional calculus in finite type I von Neumann algebras*
16:10	**Last Coffee (Room 127)**	

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Abstracts

Application of the fixed point approach to hyperstability of some functional equation

Anna Bahyrycz

We present some hyperstability results for the functional equation

$$\sum_{i=1}^{m} A_i f(\sum_{j=1}^{n} a_{ij} x_j) + A = 0,$$

in the class of functions f mapping a normed space into a normed space (both over the field $\mathbb{F}$ of real or complex numbers) where $A, a_{ij} \in \mathbb{F}$, $A_i \in \mathbb{F} \setminus \{0\}$, $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$.

We obtain hyperstability for particular functional equations of this type and different control functions. We also provide some applications of those outcomes for characterizations of inner product spaces.

The main result has been obtained by a fixed point method.

Joint work with Jolanta Olko.

The Bergman kernel function for some Reinhardt domains

Tomasz Beberok

In 1921, S. Bergman introduced a kernel function, which is now known as the Bergman kernel function. It is well known that there exists a unique Bergman kernel function for each bounded domain in $\mathbb{C}^n$. Computation of the Bergman kernel function by explicit formulas is an important research direction in several complex variables. For which domains can the Bergman kernel function be computed by explicit formulas? The goal of this talk is to give explicit formulas for the Bergman kernel function for some Reinhardt domains. We also investigate hypergeometric functions which are needed to compute the Bergman kernel.

Ulam's type stability for linear equations of higher orders

Janusz Brzdęk

Some basic definitions and results connected with the notion of Ulam's type stability will be given. Next, general methods for investigations of that stability for the linear (difference, differential, functional and integral) equations of higher orders will be presented. They can be expressed in the terms of fixed points in suitable function spaces.

On the decomposition and the model for commuting isometries

Zbigniew Burdak

The classical Wold result gives a decomposition of an isometry into a unitary operator and a unilateral shift of certain multiplicity. As a consequence a geometrical model of an isometry is known. There is no simple generalization for a pair of isometries. By a canonical Wold decomposition of a pair of isometries we understand a decomposition into: a pair of unitary operators, a pair of unilateral shifts and two mixed pairs. A canonical Wold decomposition of a pair of isometries holds with a strong doubly commutativity assumption. Moreover, for such a pair, a geometrical model is given. For only commuting pairs a functional model or Wold type decomposition (other than a canonical one) have been proposed by several authors.

The main result of the talk concerns compatible pairs of isometries; the much bigger class than doubly commuting pairs of isometries. A decomposition and a geometrical model of a compatible pair is fully described. General pairs of commuting isometries are also considered. A class of pairs of isometries having a canonical Wold decomposition other than doubly commuting pairs is given. The description of a unitary part in a completely non doubly commuting pair of isometries is presented.

The talk is based on joint work with Marek Kosiek and Marek Słociński.

Kernels of Toeplitz operators, maximal functions and model spaces

M. Cristina Câmara

We study model spaces in the Hardy spaces H_p with $1 < p < \infty$, focusing in particular on their relation with the notion of a minimal T-kernel, and we present some decomposition results for kernels of Toeplitz operators in terms of model spaces.

This talk is based on joint work with M. T. Malheiro and J. R. Partington.

Boundedness of some operators on spaces of holomorphic functions

Isabelle Chalendar

We explain how the boundedness of a weighted composition operator on the Hardy-Hilbert space on the disc or half-plane implies its boundedness on a class of related space, including weighted Bergman spaces. This generalizes some known results for Toeplitz operators and composition operators. The methods used involve the study of lower-triangular and causal operators. The characterization of strongly continuous semigroups of operators on weighted Bergman spaces can also be derived.

Approximate inner product spaces

Jacek Chmieliński

Let $(X, \|\cdot\|)$ be a real normed space. If $\|\cdot\|_i$ is an equivalent norm coming from an inner product (cf. [2]), then the original norm $\|\cdot\|$ satisfies an approximate parallelogram law. We prove that, in some cases, the reverse is also true. In particular, we prove that if

$$\big|\, \|x+y\|^2 + \|x-y\|^2 - 2\|x\|^2 - 2\|y\|^2 \,\big| \le \varepsilon\|x-y\|^2, \qquad x, y \in X$$

with $\varepsilon \in [0,1)$, then X admits an equivalent inner product norm.

[1] J. Chmieliński: Normed spaces equivalent to inner product spaces and stability of functional equations, *Aequationes Math.*, 87 (2014), 147–157.

[2] J.T. Joichi: Normed linear spaces equivalent to inner product spaces, *Proc. Amer. Math. Soc.* 17 (1966), 423–426.

On spectral analysis of block Jacobi matrices

Petru Cojuhari

We propose to discuss spectral properties of operators generated by block Jacobi type matrices. Emphasis is placed on describing of spectra of considered operators. Applications to perturbed periodic Jacobi matrices will be considered. In particular, estimate formulae for the number of the eigenvalues created by perturbations in the gaps of the unperturbed operator will be presented.

Berger measures for transformations of subnormal weighted shifts

Raúl Curto

A subnormal weighted shift may be transformed to another shift in various ways, such as taking the p-th power of each weight or forming the Aluthge transform. We determine in a number of cases whether the resulting shift is subnormal, and, if it is, we find a concrete representation of the associated Berger measure.

We do this directly for finitely atomic measures, and using both Laplace transform and Fourier transform methods for more complicated measures. Alternatively, the problem may be viewed in purely measure theoretic terms as the attempt to solve moment matching equations such as $(\int t^n \, d\mu(t))^2 = \int t^n \, d\nu(t)$ $(n = 0, 1, \ldots)$ for one measure given the other.

The talk is based on joint work with George R. Exner (Bucknell Univ., USA).

Removing zero from the numerical range

Cristina Diogo

Let $\mathcal{T} \subseteq \mathcal{B}(H)$ be a given non-empty set of operators on a separable complex Hilbert space H. In this talk we will discuss for which operators $A \in \mathcal{B}(H)$, with 0 in $\overline{W(A)}$, the closure of the numerical range of A, there exists $T \in \mathcal{T}$ such that $0 \notin \overline{W(TA)}$. Some particular sets of operators $\mathcal{T}$ will be considered. For instance, the set of positive operators, the set of selfadjoint operators, and the set of operators with zero in the closure of the numerical range.

The talk is based on joint work with Janko Bračič.

On semigroups of nonnegative functions and positive operators

Roman Drnovšek

We give extensions of results on nonnegative matrix semigroups which deduce finiteness or boundedness of such semigroups from the corresponding local properties, e.g., from finiteness or boundedness of values of certain linear functionals applied to them. We also consider more general semigroups of nonnegative functions.

The talk is based on joint work with Heydar Radjavi (University of Waterloo, Canada).

The LQ–problem for infinite–dimensional systems with bounded operators

Piotr Grabowski

Let H, U and Y be Hilbert spaces with scalar products $\langle\cdot,\cdot\rangle_{\mathrm{H}}$, $\langle\cdot,\cdot\rangle_{\mathrm{U}}$ and $\langle\cdot,\cdot\rangle_{\mathrm{Y}}$, respectively. Our aim in this paper is to minimize the quadratic performance index

$$J(x_0,u)=\int_0^\infty \begin{bmatrix} y(t)\\ u(t)\end{bmatrix}^* \begin{bmatrix} Q & N\\ N^* & R\end{bmatrix} \begin{bmatrix} y(t)\\ u(t)\end{bmatrix} \mathrm{d}t \ , \tag{1}$$

where $Q=Q^*\in\mathbf{L}(\mathrm{Y})$, $N\in\mathbf{L}(\mathrm{U},\mathrm{Y})$ and $R=R^*\in\mathbf{L}(\mathrm{U})$, over trajectories of the system

$$\left\{\begin{array}{rcl} \dot{x}(t) &=& Ax(t)+Bu(t)\\ x(0) &=& x_0\\ y(t) &=& Cx(t)\end{array}\right\}, \qquad t\geq 0 \ . \tag{2}$$

where (A,B,C) is a triple of bounded operators $A\in\mathbf{L}(\mathrm{H})$, $B\in\mathbf{L}(\mathrm{U},\mathrm{H})$, $C\in\mathbf{L}(\mathrm{H},\mathrm{Y})$.

The following result, generalizing that of [2], will be proved, discussed and illustrated by an example. In particular, we shall show how this problem is related with an LQ–problem for system having unbounded state, control and observation operators [1].

Theorem 1 If the *observability map*: $(\Psi x_0)(t):=Ce^{(\cdot)A}x_0$ belongs to $\mathbf{L}(\mathrm{H},\mathrm{L}^2(0,\infty;\mathrm{Y}))$, the *input-output map* $(\mathbb{F}u)(t):=\int_0^t Ce^{(t-\tau)A}Bu(\tau)\mathrm{d}\tau$ is in $\mathbf{L}(\mathrm{L}^2(0,\infty;\mathrm{U}),\mathrm{L}^2(0,\infty;\mathrm{Y}))$ and there exists $\varepsilon>0$ such that the *Popov spectral function* is coercive, i.e.,

$$\Pi(j\omega):=R+2\,\mathrm{Re}\left[N^*\hat{G}(j\omega)\right]+\left[\hat{G}(j\omega)\right]^*Q\hat{G}(j\omega)\geq\varepsilon I \quad \text{a.e. on} \ \ j\mathbb{R} \ ,$$

where $\hat{G}(s):=C(sI-A)^{-1}B$, then the problem has a unique solution. The optimal control $u^c\in\mathrm{L}^2(0,\infty;\mathrm{U})$ can be realized in the linear feedback form

$$u^c(t)=-R^{-1}\left[B^*\mathcal{H}+N^*C\right]x^c(t) \ , \tag{3}$$

where $\mathcal{H}$ stands for the minimal cost operator, and $\mathcal{H}$ satisfies the *operator Riccati equation*

$$A^*\mathcal{H}+\mathcal{H}A+C^*QC=(\mathcal{H}B+C^*N)R^{-1}(B^*\mathcal{H}+N^*C) \ . \tag{4}$$

[1] P. Grabowski, *The lq–controller synthesis problem for infinite–dimensional systems in factor form*, OPUSCULA MATHEMATICA, **33** (2013), pp. 29-79.

[2] J.C. Oostveen. R.F. Curtain, *Riccati equations for strongly stabilizable bounded linear systems*, Automatica, **34** (1998), 953-967.

On the C*-algebra generated by Toeplitz operators and Fourier multipliers on the Hardy space of a locally compact group

Uğur Gül

Let G be a locally compact abelian Hausdorff topological group which is non-compact and whose Pontryagin dual Γ is partially ordered. Let $\Gamma^+ \subset \Gamma$ be the semigroup of positive elements in Γ. The Hardy space $H^2(G)$ is the closed subspace of $L^2(G)$ consisting of functions whose Fourier transforms are supported on Γ^+. In this paper we consider the C*-algebra $C^*(\mathcal{T}(G) \cup F(C(\dot{\Gamma^+})))$ generated by Toeplitz operators with continuous symbols on G which vanish at infinity and Fourier multipliers with symbols which are continuous on one point compactification of Γ^+ on the Hilbert-Hardy space $H^2(G)$. We characterize the character space of this C*-algebra using a theorem of Power.

Numerical Ranges and Random Operators

Raffael Hagger

After the introduction of random matrices to nuclear physics by Eugene Wigner in 1955, random quantum systems have grown in popularity. Examples include vortex line pinning in superconductors, quantum billiards and disordered crystals. Of particular interest are, of course, the spectra of the corresponding Hamiltonians. In this talk we consider random (non-self-adjoint) tridiagonal operators on $\ell^2(\mathbb{Z})$. It turns out to be very difficult to compute the spectrum even in simple cases like the Feinberg-Zee random hopping model, which serves as our main example. For the numerical range, however, the story is much easier. Indeed, the numerical range of any random tridiagonal operator can be expressed as the convex hull of certain ellipses. Furthermore, the numerical range equals the convex hull of the spectrum in this case. Thus it provides the best convex approximation of the spectrum. We conclude with an idea of how to improve this upper bound even further and show first results for the Feinberg-Zee random hopping matrix and related operators.

[1] R. Hagger: *On the Spectrum and Numerical Range of Random Tridiagonal Operators*, in preparation.

Inverse spectral problems for energy-dependent Sturm–Liouville equations

Rostyslav Hryniv

The aim of the talk is to study the direct and inverse spectral problems for energy-dependent Sturm–Liouville equations arising in many models of classical and quantum mechanics. In contrast to the classical case, energy-dependent Sturm–Liouville problems with real-valued potentials can possess nonreal and nonsimple spectra. We give a complete characterization of their spectra and suitably defined norming constants and then solve the inverse problem of reconstructing energy-dependent Sturm–Liouville equations from either two spectra or one spectrum and the sequence of the norming constants. The approach is based on connection between the spectral problems under consideration and those for Dirac operators of special form.

The talk is based on joint work with N. Pronska (Lviv, Ukraine).

[1] R. Hryniv and N. Pronska: Inverse spectral problem for energy-dependent Sturm–Liouville equation, *Inverse Probl.*, 28 (2012), id. 085008.

[2] N. Pronska: Reconstruction of energy-dependent Sturm–Liouville equations from two spectra, *Integr. Equat. Oper. Th.*, 76 (2013), 403–419.

Unbounded Composition Operators in L^2-Spaces

Zenon Jabłoński

Let us denote by $L^2(\mu) = L^2(X, \mathcal{A}, \mu)$ the Hilbert space of all square integrable complex functions on X over a σ-finite measure space and let ϕ be an $\mathcal{A}$-measurable transformation of X. Denote by $\mu \circ \phi^{-1}$ the positive measure on $\mathcal{A}$ given by $\mu \circ \phi^{-1}(\Delta) = \mu(\phi^{-1}(\Delta))$ for all $\Delta \in \mathcal{A}$. We say that ϕ is *nonsingular* if $\mu \circ \phi^{-1}$ is absolutely continuous with respect to μ. It is easily seen that if ϕ is nonsingular, then the mapping $C_\phi \colon L^2(\mu) \supseteq \mathcal{D}(C_\phi) \to L^2(\mu)$ given by

$$\mathcal{D}(C_\phi) = \{f \in L^2(\mu) \colon f \circ \phi \in L^2(\mu)\} \text{ and } C_\phi f = f \circ \phi \text{ for } f \in \mathcal{D}(C_\phi),$$

is well-defined and linear. Such an operator is called a *composition operator* induced by ϕ. The lecture will be a survey of results concerning unbounded composition operators on $L^2(\mu)$. The results presented here are contained in [1].

[1] P. Budzyński, Z. J. Jabłoński, I. B. Jung, J. Stochel, *On unbounded composition operators in L2-spaces*, Ann. Mat. Pur. Appl. **193** (2014), 663-688.

Crossed products by Hilbert pro-C^*-bimodules

Maria Joita

The crossed product of a pro-C^*-algebra A by a Hilbert pro-C^*-bimodule (X, A) is the pro-C^*-algebra $A \times_X \mathbb{Z}$ generated by the universal covariant representation of (X, A). We show that it can be realized as an inverse limit of crossed products of C^*-algebras by Hilbert C^*-bimodules. We also show that if the Hilbert (pro-) C^*-bimodules (X, A) and (Y, B) are isomorphic, then $A \times_X \mathbb{Z}$ and $B \times_Y \mathbb{Z}$ are isomorphic as (pro-) C^*-algebras, and we prove a property of "associativity" between " $\otimes_{\min}$ " and " $\times_X$ ".

The talk is based on joint work with I. Zarakas (University of Athens, Greece).

Functional commutant of asymptotically cyclic contractions

László Kérchy

Let $\mathcal{H}$ be a separable complex Hilbert space. We recall that $\mathcal{L}_1(\mathcal{H})$ is the set of those contractions T acting on $\mathcal{H}$, which are completely quasianalytic (i.e., the quasianalytic spectral set $\pi(T)$ coincides with the whole unit circle $\mathbb{T}$) and asymptotically cyclic (i.e., the unitary asymptote of T is cyclic). The study of this class is motivated by the hyperinvariant subspace problem; see [1]. For any $T \in \mathcal{L}_1(\mathcal{H})$, the commutant $\{T\}'$ can be identified with a subalgebra $\mathcal{F}(T)$ of $L^\infty(\mathbb{T})$ containing H^∞. We know that $\mathcal{F}(T)$ is quasianalytic, that is $f(\zeta) \neq 0$ for a.e. $\zeta \in \mathbb{T}$, whenever f is a non-zero element of $\mathcal{F}(T)$. We are interested in the following questions. Which function algebras $H^\infty \subset \mathcal{A} \subset \mathcal{L}^\infty(\mathbb{T})$ are attainable as a functional commutant: $\mathcal{A} = \mathcal{F}(T)$, and what kind of information can be derived from the properties of $\mathcal{F}(T)$ on the behaviour of T? It is known that if $\mathcal{F}(T) \neq H^\infty$, then the closure $\mathcal{F}(T)^-$ is a Douglas algebra (i.e., $\mathcal{F}(T)^- = (\overline{\mathcal{B}}H^\infty)^-$ with a semigroup $\mathcal{B}$ of inner functions) containing $H^\infty + C(\mathbb{T})$, hence $\mathcal{F}(T)^-$ is not quasianalytic. The pre-Douglas algebra $\overline{\mathcal{B}}H^\infty$ is obviously quasianalytic. Answering a question posed in [1] we show that H^∞ is the only attainable pre-Douglas algebra. We give example for the case when $\mathcal{F}(T) \neq H^\infty$ and $\mathcal{F}(T)$ does not contain the conjugate of any non-constant inner function. The similarity invariance of $\mathcal{F}(T)$ is shown. Analytic contractions and weighted bilateral shifts are considered as illuminating examples.

The talk is based on a joint work with A. Szalai, accepted for publication in the *Studia Mathematica*.

[1] L. Kérchy, Quasianalytic contractions and function algebras, *Indiana Univ. Math. J.*, **60** (2011), 21–40.

C-symmetric operators and reflexivity

Kamila Kliś-Garlicka

Let C be an isometric antilinear involution in a Hilbert space $\mathcal{H}$. A bounded operator $T \in B(\mathcal{H})$ is called *C-symmetric*, if $CTC = T^*$. Let us denote the set of all C-symmetric operators by $\mathcal{C}$.

We will describe the preanihilator of $\mathcal{C}$. Moreover, we will show that the subspace of all C-symmetric operators is transitive, 2-reflexive and even 2-hyperreflexive.

The talk is based on joint work with Marek Ptak.

Minimal projections in three-dimensional normed spaces

Tomasz Kobos

By a result of Bohnenblust for every three-dimensional normed space X and its two-dimensional subspace Y, there exists a projection $P : X \to Y$ such that $||P|| \leq \frac{4}{3}$. The aim of the talk is to give a sketch of the proof of the following theorem: if for some subspace Y the minimal projection $P : X \to Y$ satisfies $||P|| \geq \frac{4}{3} - R$ for some $R > 0$, then there exists two dimensional subspace Z of X and projection $Q : X \to Z$ for which $||Q|| \leq 1 + \Phi(R)$ where $\Phi(R) \to 0$ as $R \to 0$. In other words, every space which has a subspace of almost maximal projection constant has also a subspace of almost minimal projection constant. As a consequence, every three-dimensional space has a subspace with the projection constant strictly less than $\frac{4}{3}$, which gives a non-trivial upper bound for the problem posed by Bosznay and Garay. We shall also characterize all three-dimensional spaces which have a subspace with the projection constant equal to $\frac{4}{3}$.

Szegö type decompositions of isometries

Marek Kosiek

Let V be an isometry on a Hilbert space $\mathcal{H}$. By its elementary measure μ_x for $x \in \mathcal{H}$ we mean the mapping $\mathcal{B} \ni \sigma \to \langle E(\sigma)x, x\rangle$, where E denotes the spectral measure of its minimal unitary extension and $\mathcal{B}$ the collection of all Borel subsets of its spectrum.

A decomposition $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$, reducing for V, we call Szegö type decomposition, if all elementary measures μ_x are for $x \in \mathcal{H}_1$ Szegö singular, and the subspace $\mathcal{H}_2$ is spanned by vectors which elementary measures are Szegö measures.

We present two different Szegö type decompositions for Hilbert space isometries. For unitary operators they are identical.

The talk is based on joint work with Zbigniew Burdak, Patryk Pagacz and Marek Słociński.

Generalized Markov properties

Agnieszka Kowalska

In 1889 A. Markov proved that for every polynomial p in one variable $\|p'\|_{[-1,1]} \leq (\deg p)^2 \|p\|_{[-1,1]}$, where $\|p\|_I = \sup |p|(I)$. Markov's inequality for derivatives of polynomials and its generalizations are still the subject of many investigations. The reason lay with its numerous applications in different domains of mathematics and physics. We are interested in finding generalizations of this inequality for subsets of algebraic sets. The talk is based on joint work with Mirosław Baran.

On S-matrix of Schrödinger Operators with Non-Symmetric Zero-Range Potentials

Sergii Kuzhel

The key point of Pseudo-Hermitian Quantum Mechanics (PHQM) is the employing of non-self-adjoint operators for the description of experimentally observable data [1]. Briefly speaking, a given non-self-adjoint operator A in a Hilbert space $\mathfrak{H}$ can be interpreted as a physically meaningful Hamiltonian if its spectrum is real and there exists a new inner product that ensures the (hidden) self-adjointness of A.

As usual, in PHQM studies, a non-self-adjoint operator A admits a representation $A = A_0 + V$, where A_0 is a self-adjoint operator in the Hilbert space $\mathfrak{H}$ and V is a non-symmetric potential characterized by a set $\Upsilon = \{\varepsilon\}$ of complex parameters ε. One of important problems is the description of quantitative and qualitative changes of spectrum $\sigma(A)$ when ε runs Υ. A typical picture is the following:

$$\begin{array}{ccccc} I & & II & & III \\ \text{non-real} & \leftrightarrow & \text{spectral singularities} & \leftrightarrow & \text{similarity to} \\ \text{eigenvalues} & & \text{exceptional points} & & \text{a self-adjoint operator} \end{array}$$

An operator from the domain I cannot be realized as a self-adjoint operator for any choice of inner product. While, an operator from the domain III turns out to be self-adjoint with respect to a new inner product of $\mathfrak{H}$ which is equivalent to the initial one. The domain II can be interpreted as a boundary between I and III and the corresponding operators will keep only part of properties of I and III. For instance, if an operator A corresponds to II, then its spectrum is real (similarly to III) but A cannot be made self-adjoint by an appropriative choice of equivalent inner product of $\mathfrak{H}$ (in spirit of I). This phenomenon deals with the appearing of 'wrong' spectral points of A which are impossible for the spectra of self-adjoint operators. Traditionally, these spectral points are called *exceptional points* if they are located at the discrete spectrum of A and *spectral singularities* in the case of the continuous spectrum.

It is naturally to suppose that the evolution of spectral properties discussed above can be described in terms of poles of S-matrices of non-self-adjoint operators A. We check this hypothesis by considering the set of operators A generated by the Schrödinger type differential expression $A = -\frac{d^2}{dx^2} + V$ with non-symmetric zero-range potential

$$V = a < \delta, \cdot > \delta(x) + b < \delta', \cdot > \delta(x) + c < \delta, \cdot > \delta'(x) + d < \delta', \cdot > \delta'(x),$$

where δ and δ' are, respectively, the Dirac δ-function and its derivative and a, b, c, d are complex numbers.

We show that poles of S-matrix $\mathsf{S}(\cdot)$ *completely characterize* the properties of Schrödinger operators A with non-symmetric zero-range potentials. Precisely, poles of $\mathsf{S}(\cdot)$ on the physical sheet $\mathbb{C}_+$ describe the discrete spectrum σ_p of A. The appearance of exceptional points on σ_p is distinguished by poles of order 2 on the physical sheet. The existence of spectral singularities on the continuous spectrum of A is determined by poles of $\mathsf{S}(\cdot)$ on the extended real line $\overline{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$. The property of similarity of A to a self-adjoint operator means that the S-matrix $\mathsf{S}(\cdot)$ has poles in the nonphysical sheet $\mathbb{C}_-$ or $\mathsf{S}(\cdot)$ has simple non-zero imaginary poles.

The talk is based on joint work with P. A. Cojuhari and A. Grod.

[1] C. M. Bender: Making sense of non-Hermitian Hamiltonians, *Rep. Prog. Phys.*, 70 (2007), 947–1018.

Measures, dilations, and frames

David Larson

We will discuss some recent work of the speaker on the interaction between operator algebras on the one hand and the frame and basis theory of applied harmonic analysis on the other hand. This represents ongoing joint work with Deguang Han, Bei Liu and Rui Liu. In a recent article (AMS memoir) we developed a general dilation theory for operator valued measures and maps between von Neumann algebras that include measures and maps that are not necessarily completely bounded. In a subsequent article (JFA) we obtained some results for dilations of operator-valued systems of imprimitivity. Our main results state that any operator-valued measure, not necessarily completely bounded, always has a dilation to a projection-valued measure acting on a Banach space, and every bounded linear map (again not necessarily completely bounded) on a Banach algebra has a bounded homomorphism dilation acting on a Banach space. Here the dilation space often needs to be a Banach space even if the underlying space is a Hilbert space, and the projections are idempotents that are not necessarily self-adjoint. These results lead to some new connections between frame theory and operator algebras that we view as “noncommutative" frame theory. We will also discuss some joint work with Franek Szafraniec on abstract dilations of maps.

On generating solutions of a generalized Volterra integral equation

Zbigniew Leśniak

We consider the following integral equation

$$\psi(x) = \int_a^x N(x, t, \psi(\alpha(x, t)))\, dt + G(x), \qquad x \in I,$$

for continuous ψ mapping I into B, where I is a real interval of the form $[a, \infty)$ or $[a, b]$ or $[a, b)$ with some real $a < b$, $\int$ denotes the Bochner integral, and $G : I \to B$, $N : I \times I \times B \to B$ and $\alpha : I \times I \to I$ are given continuous functions. Using an operator acting on the space of all continuous functions defined on I with values in B, we show that, under suitable assumptions, approximate solutions of this equation generate its exact solutions. We also present an application of our results to a linear Volterra integral equation of convolution type.

The talk is based on joint work with Anna Bahyrycz and Janusz Brzdęk.

Weighted shifts acting on directed semi-trees

Witold Majdak

We extend the notion of a weighted shift acting on a directed tree ([1]) to the case of a weighted shift acting on a more general graph, which we call a directed semi-tree. We investigate properties of such operators and show that the generalized creation operator can be regarded as a weighted shift acting on a directed semi-tree. The talk is based upon a joint work with Jerzy B. Stochel.

[1] Z.J. Jabłoński, I.B. Jung, J. Stochel, Weighted shifts on directed trees, *Mem. Am. Math. Soc.* 1017 (2012), 107 pp.

Asymptotics of the discrete spectrum for complex Jacobi matrices

Maria Malejki

Spectral properties of non-symmetric tridiagonal matrices and complex Jacobi matrices have been recently investigated by several authors: Beckermann and Kaliaguine, Djakov and Mityagin, Egorova and Golinskii and others. Tridiagonal matrices are strongly connected with formal orthogonal polynomials on the complex plane, Mathieu equation and functions, and Bessel functions. Spectral properties of self-adjoint Jacobi matrices were tested using various methods. Systematic research concerning spectral properties of complex tridiagonal infinite matrices is not easy because the structure of complex sequences can be more complicated then the structure of real sequences. Moreover, the spectral theorem and its consequences fail in this case. Nevertheless, some properties of real Jacobi matrices can be carried over to the complex tridiagonal matrices. We observe that effective research methods for non-selfadjoint operators use the Riesz projections instead of the spectral theorem.

Now we are going to discuss the asymptotic formulae for the point spectrum of unbounded discrete operators given by special classes of tridiagonal complex matrices.

On the Baskakov-Durrmeyer type operators

Renata Malejki

In this paper we will study some approximate properties of Baskakov-Durrmeyer type operators. We determine the rate of convergence and prove the Voronovskaya type theorem for that operators.

The talk is based on joint work with Eugeniusz Wachnicki.

[1] Z. Ditzian: J. Approx. Theory *Direct estimate for Bernstein polynomials*, 79 (1994), 165–166.

[2] A. Erençin: Appl. Math. Comput. *Durrmeyer type modification of generalized Baskakov operators*, 218 (2011), 4384–4390.

[3] G. Krech: Appl. Math. Comput. *A note on the paper "Voronovskaya type asymptotic approximation by modified Gamma operators*, 219 (2013), 5787–5791.

[4] G. Krech, E. Wachnicki: Demonstratio Math. *Direct estimate for some operators of Durrmeyer type in exponential weighted space*, 47 (2) (2014), 336–349.

[5] R. Malejki, E. Wachnicki: Comm. Math. *On the Baskakov-Durrmeyer type operators*, 54 (1) (2014).

[6] S. M. Mazhar, V. Totik: Acta Sci. Math. *Approximation by modified Szász operators*, 49 (1985), 257–269.

[7] A. F. Timan: *Theory of Approximation of Function of a Real Variable*, 1960.

Kernels of Toeplitz operators with symbols in Q-classes of matrix functions

M. Teresa Malheiro

We generalize the notion of Q-classes C_{Q_1,Q_2}, which was introduced in the context of Wiener-Hopf factorization, by considering very general 2 X 2 matrix functions Q_1, Q_2. This allows us to use a mainly algebraic approach to obtain several equivalent representations for each class, to study the intersections of Q-classes and to explore their close connection with certain non-linear scalar equations. The results are applied to various factorization problems and to the study of Toeplitz operators with symbol in a Q-class.

The talk is based on joint work with Cristina Câmara.

Alternating projections on Hilbert spaces

Vladimir Müller

Let M_1, M_2 be subspaces of a Hilbert space H and P_{M_1}, P_{M_2} the corresponding orthogonal projections. By a classical result of von Neumann, the sequence $(P_{M_1}P_{M_2})^k x$ is convergent for each $x \in H$; the limit is equal to $P_M x$, where $M = M_1 \cap M_2$. The result was generalized by Halperin to any finite number of subspaces $M_1, \dots, M_n \subset H$ and sequence of iterations $(P_{M_1}P_{M_2}\cdots P_{M_n})^k x$.

Let $(i_k) \subset \{1, \dots, n\}$ be any sequence, $x_0 \in H$ and $x_k := P_{M_k} x_{k-1}$. By a result of I. Amemiya and T. Ando, this sequence is always weakly convergent. It was an open problem for a long time whether such a sequence must be convergent in the norm. The problem was recently answered negatively by A. Paszkiewicz, who constructed 5 orthogonal projections and a non-convergent sequence of iterations.

We show a similar example of 3 projections which fills the gap between the results of von Neumann and Paszkiewicz.

Joint work with E. Kopecka.

Functional calculus in finite type I von Neumann algebras

Piotr Niemiec

The lecture will be devoted to a new concept of extending the classical Borel functional calculus for normal operators to a functional calculus for arbitrary elements of finite type I von Neumann algebras M. More precisely, a certain class of matrix-valued Borel matrix functions will be introduced and it will be shown that all functions of that class naturally operate on any operator T in M in a way such that uniformly bounded sequences $f_1, f_2, \dots$ of functions that converge pointwise to 0 transform into sequences $f_1[T], f_2[T], \dots$ of operators in M that converge to 0 in the $*$-strong operator topology. A connection between this functional calculus and the double

$*$-commutant of a separable Hilbert space operator belonging to a finite type I von Neumann algebra shall also be discussed. If time permits, some conclusions concerning so-called operator-spectra of such operators will be drawn and a new variation of the spectral theorem for them shall be formulated.

De Branges-Rovnyak spaces and generalized Dirichlet spaces

Maria Nowak

Let T_χ denote the Toeplitz operator on H^2 whose symbol is $\chi \in L^\infty$ on the unit circle $\mathbb{T}$. For given a function b in the unit ball of H^∞, the *de Branges-Rovnyak space* $\mathcal{H}(b)$ is the image of H^2 under the operator $(I - T_b T_{\bar{b}})^{1/2}$ with the range norm.

Here we deal with the case when b is not an extreme point of the unit ball of H^∞. We describe the structure of some spaces $\mathcal{H}(b)$ and their connections with the generalized Dirichlet spaces defined below.

For $\lambda \in \mathbb{T}$ we define the local Dirichlet integral of f at λ by

$$D_\lambda(f) = \frac{1}{2\pi} \int_0^{2\pi} \left| \frac{f(\lambda) - f(e^{it})}{\lambda - e^{it}} \right|^2 dt.$$

where $f(\lambda)$ is the nontangential limit of f at λ. If $f(\lambda)$ does not exist, then we set $D_\lambda(f) = \infty$. Let μ be a positive Borel measure on $\mathbb{T}$. *The generalized Dirichlet space* $\mathcal{D}(\mu)$ consists of those functions $f \in H^2$ for which

$$D_\mu(f) = \int_{\mathbb{T}} D_\lambda(f) d\mu(\lambda) < \infty.$$

In 1997 D. Sarason showed that $\mathcal{D}(\delta_\lambda)$, where δ_λ is the unit mass at λ, can be identified with $\mathcal{H}(b_\lambda)$, where $b_\lambda(z) = (1 - w_0)\bar{\lambda} z/(1 - w_0 \bar{\lambda} z)$, and $w_0 = (3 - \sqrt{5})/2$. Further results showing connection between the spaces $\mathcal{H}(b)$ and $D(\mu)$ have been recently obtained by T. Ransford, D. Guillot, N. Chevrot and C. Costara, E. Fricain, W. Ross and others.

Joint work with Bartosz Łanucha.

Application of the fixed point approach to stability of some functional equation

Jolanta Olko

The stability of a functional equation can be formulated as follows: whether a function which satisfies the equation approximately is close in some sense to its solution. Among other tools the fixed point approach is often applied for proving stability of functional equations. Using this method, we give sufficient conditions for the Hyers-Ulam stability of a wide class of functional equations and control functions. The criterion may be used for checking stability of the particular functional equation. The results are a generalization of many the well known outcomes concerning stability as well as the recent ones.

The talk is based on joint work with Anna Bahyrycz.

An asymmetric Putnam-Fuglede theorem for paranormal and $*$-paranormal operators

Patryk Pagacz

The presentation concerns operators on a complex Hilbert space $\mathcal{H}$. An operator T is called *dominant* if and only if for all $\lambda \in \mathbb{C}$ there is $M_\lambda > 0$ such that $\|(\lambda - T)^* x\| \leq M_\lambda \|(\lambda - T)x\|$ for all $x \in \mathcal{H}$. Moreover, an operator is called $*$-*paranormal* if and only if $\|T^* x\|^2 \leq \|T^2 x\| \|x\|$ for all $x \in \mathcal{H}$

Let us recall the well-known Putnam-Fuglede theorem.

Theorem 2 *Let A, B and X be bounded operators on a Hilbert space with A, B being normal. If $AX = XB$, then $A^*X = XB^*$.*

There are many asymmetric extensions of this theorem. For example, if we assume that A, B^* are dominant then the above statement holds true (see [4]).

The main purpose of this talk is to present the following theorem.

Theorem 3 *Let A, B and X be bounded operators on a Hilbert space with A, B being $*$-paranormal. If $AX = XB^*$, then $A^*X = XB$.*

The proof of the above theorem gives also an alternative proof of the asymmetric Putnam-Fuglede theorem for dominant operators.

The talk is based on joint work with Ahmed Bachir.

[1] A. Bachir, P. Pagacz: An asymmetric Putnam-Fuglede theorem for paranormal and $*$-paranormal operators, *preprint*, 2014.

[2] S.K. Berberian: Approximate proper vectors, *Proc. Amer. Math. Soc.*, 13 (1962), 111–114.

[3] C.R. Putnam: On the normal operators on Hilbert space, *Amer. J. Math.*, 73 (1951), 357–362.

[4] J.G. Stamfli, B. Wadhwa: An asymmetric Putnam-Fuglede theorem for dominant operators, *Indian Univ. Math. J.*, 25 (1976), 359–365.

Cartan subalgebras of operator ideals

Sasmita Patnaik

Denote by $\mathcal{U}_\mathcal{I}(\mathcal{H})$ the group of all special unitary operators $V \in \mathbf{1} + \mathcal{I}$ where $\mathcal{H}$ is a separable infinite-dimensional complex Hilbert space and $\mathcal{I}$ is an ideal of $B(\mathcal{H})$. An ideal has a natural structure of a Lie algebra where the Lie bracket is defined as the commutator of operators. For every Cartan subalgebra $\mathcal{C}$ of $\mathcal{I}$ (maximal abelian self-adjoint subalgebra of $\mathcal{I}$), its conjugacy class is defined as the set of Cartan subalgebras $\{V\mathcal{C}V^* \mid V \in \mathcal{U}_\mathcal{I}(\mathcal{H})\}$. For nonzero proper ideals $\mathcal{I}$ we construct an uncountable family of Cartan subalgebras $\mathcal{C}$ of $\mathcal{I}$ with distinct conjugacy classes under the action of the group $\mathcal{U}_\mathcal{I}(\mathcal{H})$. This is in contrast to the by now classical observation of P. de La Harpe who showed that when $\mathcal{I}$ is any of the Schatten ideals, there is precisely one conjugacy class under the action of the full group of unitary operators.

The equality $C^{*2}C^2 = (C^*C)^2$ is not sufficient for quasinormality of a composition operator C in L^2-space

Pawel Pietrzycki

It is proved that a closed densely defined operator C is quasinormal if and only if the equality $C^{*n}C^n = (C^*C)^n$ holds for $n = 2, 3$. Let W be bounded injective weighted shift which satisfies the equality $W^{*n}W^n = (W^*W)^n$. We prove that operator W is then quasinormal. We will construct examples of bounded, non-quasinormal operator C which satisfies equality $C^{*n}C^n = (C^*C)^n$. An example of such a operator is given in the class of weighted shifts on directed trees. What is important, the directed tree used in the construction is rootless and therefore the operator in example is unitarily equivalent to a composition operator in L^2-space.

Stability and hyperstability of some equations

Magdalena Piszczek

Using the fixed point theorem for functional spaces we present the results concerning the stability and hyperstability of some equations. In details we discuss the general linear equation and the Drygas one.

Weighted Sub-Bergman Hilbert Spaces

Renata Rososzczuk

We consider Hilbert spaces that are counterparts of de Branges-Rovnyak spaces in the context of the weighted Bergman spaces in the unit disk A^2_α, $-1 < \alpha < \infty$ These spaces have been studied by K. Zhu (1996, 2003), S. Sultanic (2006), A. Abkar and B. Jafarzadeh (2010). We extend some results from these papers.

Joint work with Maria Nowak, Maria Curie-Sklodowska University, Lublin, Poland.

Partial order on partial isometries

William Ross

In this joint work with Stephan Garcia (Pomona College, USA) and Rob Martin (University of Cape Town, South Africa) I will discuss how one places partial orders on the set of partial isometries on a Hilbert space. Through work of Krein, one can turn this partial isometry problem into a problem of multipliers and isometric multipliers from one Hilbert space of analytic functions to another.

Analytic measures and weak-star closures in general function algebras

Krzysztof Rudol

Analytic measures, called also A-measures, or Henkin measures form a strong tool in studying algebras of analytic functions of many complex variables (and in their use in operator theory). By specifying this notion to a union $Q \subset X$ for Gleason parts of a function algebra $A \subset C(X)$ we formulate and prove an abstract version of Hekin's theorem, obtained earlier by advanced complex analytic methods in strictly pseudoconvex domains (see [3], [2]) and on polydomains [1]. Namely, we say that a complex measure μ on X *is an A-measure for the algebra A at the points of* Q if $\int f_n d\mu \to 0$ for any bounded sequence (f_n) in A converging to zero at the points of Q. If $A = A(\Omega)$ is the algebra of functions continuous on $X = \overline{\Omega}$ analytic on Ω, one takes $Q = \Omega$ and Henkin proved that any measure absolutely continuous w.r. to an A-measure shares this property. Our approach is quite general, using weak-star topology and relations between quotient norms, yielding a formulation, that works for a wider class of domains Ω, when applied to $A = H^\infty(\Omega)$, or to $A = A(\Omega)$. If only Q is a countable union of Gleason parts in the spectrum of A, we show that *any measure on X absolutely continuous w.r. to some representing measure for A at a point of Q must be an A-measure at the points of Q.*

By Lebesgue-type decompositions we can assume that Q is a single nontrivial Gleason part G. Our main result implying the mentioned extension of Henkin's theorem concerns weak-star closures. Namely, we consider $\sigma(A^{***}, A^{**})$-closures $\overline{G}^{ws}$ of G embedded in the spectrum of the bi-dual algebra A^{**} with its Arens product, the band of measures $\mathcal{M}_G$ related to G, its weak-star closure $\mathcal{M}_1 = \overline{\mathcal{M}_G}$ and the complementary band $(\mathcal{M}_1)^s$ (of measures singular w. r. to any $\nu \in \mathcal{M}_1$). We prove that always $\overline{G}^{ws}$ is closed and open in the spectrum of $C(X)^{**}$, $\mathcal{M}_1$ is the band of all measures carried by $\overline{G}^{ws}$ and $(\mathcal{M}_1)^s$ is the weak-star closure of the complementary band $(\mathcal{M}_G)^s$.

Also certain other results implied by this theorem will be presented.

The talk is based on joint work [4] with Marek Kosiek (IM UJ, Kraków).

[1] O.B.Bekken: Rational approximation on product sets, *Trans. Amer. Math. Soc.*, 191 (1974), 301–316.

[2] B.J. Cole, R.M. Range: A-measures on complex manifolds and some applications,*J. Funct. Anal* 11 (1972), 393–400.

[3] G.M Henkin: Banach algebras if analytic functions on the ball and on the bicylinder are not isomorphic, *Funkc. Analiz i Prilozh.* 2 (1968), 334–341.

[4] M.Kosiek, K. Rudol: Dual algebras and A-measures (in preparation)

Subnormailty of unbounded composition operators in L^2 spaces

Jan Stochel

The aim of my talk is to present some recent achievements in the study of unbounded composition operators in L^2 spaces over σ-finite measure spaces. The main question I intend to discuss is subnormality of such operators. In general, there is no satisfactory description of subnormality of unbounded Hilbert space operators. In the case of bounded composition operators, it is characterized by the celebrated Lambert's condition, which does not work in the unbounded case. The solution proposed in my talk is of different nature, namely it depends on the existence of a family of probability measures on the closed half line satisfying the consistency condition. This new criterion becomes a necessary and sufficient condition for subnormality of bounded composition operators. Some particular instances of composition operators are discussed as well.

The talk is based on two papers written in collaboration with P. Budzyński, Z. J. Jabłoński and I. B. Jung.

[1] P. Budzyński, Z. J. Jabłoński, I. B. Jung, J. Stochel, On unbounded composition operators in L^2-spaces, *Ann. Mat. Pur. Appl.* **193** (2014), 663-688.

[2] P. Budzyński, Z. J. Jabłoński, I. B. Jung, J. Stochel, Unbounded subnormal composition operators in L^2-spaces, arXiv:1303.6486.

Relationship between the holes of the supports of representing measures of Stieltjes moment sequences and their roots

Jerzy Bartłomiej Stochel

Stieltjes moment sequences $\{a_n\}_{n=0}^{\infty}$ whose κth roots $\{\sqrt[\kappa]{a_n}\}_{n=0}^{\infty}$ are Stieltjes moment sequences are studied (κ is a fixed integer greater than or equal to 2). A formula connecting the closed supports of representing measures of $\{a_n\}_{n=0}^{\infty}$ and $\{\sqrt[\kappa]{a_n}\}_{n=0}^{\infty}$ is established. The relationship between the holes of the supports of these measures is investigated. The talk is based on a joint work with Jan Stochel ([1]).

[1] Jan Stochel, Jerzy B. Stochel, On the κth root of a Stieltjes moment sequence, *J. Math. Anal. Appl.* 396 (2012), 786–800.

Spectral behaviour of quasianalytic contractions

Attila Szalai

We pose, and answer partially, questions in connection with the spectral behaviour of quasianalytic contractions. These problems are related to the hyperinvariant subspace problem in the class of asymptotically non-vanishing contractions.

The talk is based on joint work with L. Kérchy.

On the solvability of Dirichlet problem for the weighted p-Laplacian

Ewa Szlachtowska

We are concerned with the existence and uniqueness of the weak solution for the weighted p-Laplacian. The purpose is to discuss in some depth the problem of solvability of Dirichlet problem. The main result is the existence and uniqueness of the weak solution for the Dirichlet problem provided that the weights are bounded. Furthermore, under this assumption the solution belongs to the Sobolev space $W_0^{1,p}(\Omega)$.

Joint work with Dominik Mielczarek.

Tween mappings between multiplicative subsets of function algebras and their spectral properties

Thomas Tonev

We characterize pairs of mappings $T_1, T_2 \colon \mathcal{S} \to \mathcal{T}$, not necessarily linear, between multiplicative subsets, $\mathcal{S} \subset A, \mathcal{T} \subset B$ of function algebras, A, B, on locally compact Hausdorff spaces, so that the peripheral spectra of their products posses the properties $\sigma_\pi(T_1(f)T_2(g)) \subset \sigma_\pi(fg)$ and $\sigma_\pi(T_1(f)T_2(g)) \cap \sigma_\pi(fg) \neq \varnothing$, $f, g \in \mathcal{S}$. Under natural conditions on $\mathcal{S}$, $\mathcal{T}$ we show that in these cases there is a homeomorphism $\phi \colon \delta B \to \delta A$ and a continuous function $\alpha \colon \delta B \to \mathbb{C} \setminus \{0\}$, where $\delta A, \delta B$ are the corresponding Choquet boundaries of A and B, so that $T_1(f)(y) = \alpha(y) f(\phi(y))$ and $T_2(g)(y) = \alpha(y)^{-1} g(\phi(y))$ for all $f, g \in \mathcal{S}$ and $y \in \delta B$. A large number of

previous results about mappings between function algebras subject to various spectral conditions are consequences of these results.
The talk is based on a joint work with T. Miura.

On decay of operator semigroups

Yuri Tomilov

The talk will concern an interplay between several areas of analysis arising in the study of partial differential equations. We intend to present a theory of fine scales of decay rates for operator semigroups. The theory contains, unifies, and extends several notable results on asymptotics of of operator semigroups and yields a number of new ones. Its core is a new operator-theoretical method of deriving rates of decay combining ingredients from functional calculus, and complex, real and harmonic analysis. In this talk, we will offer a glimpse at the theory.

This is joint work with Charles Batty and Ralph Chill to appear in J. Europ. Math. Soc.

Indefinite Sturm-Liouville problems for the p-Laplacian

Bruce A. Watson

The (λ, μ) eigencurves are studied for a weighted quasi-linear Sturm-Liouville-type problem of the form

$$-\Delta_p y = (p-1)(\lambda r - q - \mu s)\mathrm{sgn} y|y|^{p-1}, \quad \text{on} \ \ (0,1)$$

with Sturmian-type boundary conditions (Δ_p being the p-Laplacian). Here s is positive definite while r may change sign. Asymptotics including existence of oblique asymptotes are considered. These results are applied to obtain eigenvalue asymptotics and an oscillation theory for the single parameter eigenvalue problem where $\mu = 0$.

This joint work with Paul Binding and Patrick Browne.

On a restriction of an operator to an invariant subspace

Paweł Wójcik

For Banach spaces we consider the bounded linear operators which are surjective and noninjective. The aim of this report is to discuss an invariant subspace of some surjective operators. We show some general properties of such mappings. We examine whether such operators can restrict to an involution or a projection. Thus, we will obtain the invariant subspaces for those operators.

[1] P. Wójcik: On some restrictions of an operator to an invariant subspace, *Linear Algebra Appl.*, 450 (2014), 1–6.

Operator Theory in the Hardy space over the Bidisk

Rongwei Yang

Unilateral shift is a fundamental example for non-selfadjoint operator theory. Its invariant subspaces were characterized by Beurling's theorem in 1949 through inner functions in the Hardy space over the unit disk $H^2(\mathbb{D})$. The theorem motivated great developments in function theory and operator model theory in the following decades. This success led people to ponder if similar progress can be made for the bi-shift in the Hardy space over the bidisk $H^2(\mathbb{D}^2)$. In the late 1950s

W. Rudin construct two pathological examples which demonstrated the extreme complexity of this problem. However, despite its complexity, great progresses have been made in the past 20 years. This talk is a brief survey of this progress.

A spectral representation for a class of non-self-adjoint banded Jacobi matrices

Ewelina Zalot

The aim of this talk is to show that each non-self-adjoint Laurent operator arising from a generalized Jacobi-type matrix has a spectral-type integral representation under the assumption that its spectrum is the closure of a finite family of smooth non-intersecting curves on the complex plane. Our results extend those obtained by P.B. Naïman in [1] for tridiagonal periodic Jacobi matrices. The talk is based upon a joint work with Witold Majdak.

[1] P.B. Naïman, On the spectral theory of the non-symetric periodic Jacobi matrices, *Notes of the Faculty of Math. and Mech. of Kharkov's State University and of Kharkov's Math. Society* 30 (1964), 138–151 [in Russian].

Approximation of large eigenvalues of unbounded Jacobi matrices by eigenvalues of finite submatrices

Lech Zielinski

Infinite symmetric tridiagonal matrices called Jacobi matrices have been investigated in many recent papers in relation with some applications of quantum physical models. In particular it is important to analyse the influence of physical parameters on the discrete spectrum of Hamiltonians which appears in the asymptotic behaviour of large eigenvalues. The closely related problem of the numerical approximation of large eigenvalues of an infinite matrix by eigenvalues of its finite submatrices is usually treated by the the well-known Rayleigh-Ritz method (cf. H. Volkmer, Constr. Approx. 20, 2004, 39-54). The aim of this talk is to present an alternative method based on the decay of the resolvent kernels. This method allows us to obtain results on the approximation of eigenvalues for more general classes of Jacobi matrices and finite difference symmetric operators. More precisely we consider the operators acting in l^2 and defined by a matrix with a finite number of diagonals. The entries outside the main diagonal are assumed to be small with respect to the entries of the main diagonal and the entries of the main diagonal are assumed to tend to infinity.