Block Jacobi matrices and self-adjoint finitely cyclic operators

Marcin Moszyński (University of Warsaw, Warsaw, Poland)

If A is a linear operator, possibly unbounded, in a normed space X, then $\varphi \in X$ is *a cyclic vector (for A)* when $\varphi \in \mathrm{Dom}(A^\infty) := \bigcap_{n\in\mathbb{N}_0} \mathrm{Dom}(A^n)$, and the linear span of its orbit $\{A^n\varphi : n \in \mathbb{N}_0\}$ is dense in X (here $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$). We call A *cyclic*, if there exists a cyclic vector for A. It is a well-known result that each self-adjoint cyclic operator in an infinite dimensional Hilbert space is unitary equivalent to an operator J in the space $\ell^2(\mathbb{N})$ given by a scalar Jacobi matrix on a domain $\mathrm{Dom}(J) \supset \ell_{\mathrm{fin}}(\mathbb{N})$, where $\ell_{\mathrm{fin}}(\mathbb{N})$ is the space of the so called "finite sequences" (i.e., sequences with only finite number of non-zero terms).

My talk is devoted to the possibility of some generalizations of the above cyclic case result to the case of *finite cyclicity*, when we consider the orbit of a finite set of vectors instead of the single vector φ (or, equivalently, the orbit of a finite dimensional linear subspace). Each self-adjoint operator given by a "classical" block Jacobi matrix with all the blocks of a constant $d \times d$ size (see [1]) is a typical example of self-adjoint finitely cyclic operator. It remains also true when we consider somewhat more general sizes of the blocks ("generalized block Jacobi matrix") instead of the classical block case.

I will discuss some problems concerning the unitary equivalence of self-adjoint finitely cyclic operator to some operator given by block Jacobi matrices – generalized or classical. Some of them can be solved with quite elementary methods, while some seem to be still open problems.

[1] Ju. M. Berezanskii: *Expansions in eigenfunctions of selfadjoint operators.* Translations of Mathematical Monographs, Vol. 17, American Mathematical Society, Providence, R.I., 1968 (translated from the Russian by R. Bolstein, J. M. Danskin, J. Rovnyak and L. Shulman).