6^{th} Summer Workshop on Operator Theory

9–13 July, 2018
Kraków, Poland

The Workshop is organized by:

Department of Applied Mathematics
University of Agriculture in Krakow

Scientific Committee (Local):

Mirosław Baran (University of Agriculture in Krakow)

Petru A. Cojuhari (AGH University of Science and Technology, Krakow)

Jan Janas (Polish Academy of Sciences, Krakow)

Marek Ptak (University of Agriculture in Krakow)

Jan Stochel (Jagiellonian University in Krakow)

Franciszek H. Szafraniec (Jagiellonian University in Krakow)

Organizing Committee (University of Agriculture in Krakow):

Tomasz Beberok

Piotr Budzyński

Zbigniew Burdak

Piotr Dymek

Kamila Kliś-Garlicka

Marta Majcherczyk

Kamila Piwowarczyk

Artur Płaneta

The conference will take place in the building of the University of Agriculture in Krakow (al. Mickiewicza 24/28). Registration will be opened on Monday from 8.00 till 8.45 in the same building on the first floor. The opening of the conference will start at 9.00 in the room 120. Plenary talks will be held in the room 120, parallel sessions in rooms 120, 125.

An excursion to Pieniny is planned for Thursday (12$^{\text{th}}$ July). A bus will leave at 11.15 from the Ingardena street.

The conference dinner will be held at Krzywaczka just after the excursion (around 19.00). A bus for people not attending the excursion will leave at 18.00 from the parking in front of the Conference building. We plan to return to Krakow around 23.00. In the case of the bad weather condition foreseen the plan for Thursday will be shifted to Wednesday.

There is a wireless connection available. To get the access choose network called *WISIG* and enter the first password: *Ur2014!@#*. Then, you have to open any web page. You will be asked to enter another login and password. The login is *guest* and a password is *Ur2014#@!*.

Monday, July 9th

Conference Hall: al. Mickiewicza 24/28

8:00 – 8:45	**Registration**
9:00 – 9:20	**Opening**
Plenary session (Room 120) Chair: Vladimír Müller	
9:20 – 10:00	**William Ross** Birkhoff-James Inner functions and zero sets of analytic functions
10:00 – 10:30	**Coffee break (Room 127)**
10:30 – 11:10	**Dan Timotin** Beyond truncated Toeplitz operators
11:15 – 11:55	**Dijana Ilišević** On isometric roots of isometries
11:55 – 14:30	**Lunch**

Monday, July 9th

Conference Hall: al. Mickiewicza 24/28

Parallel sessions		
	Session A (Room 120) Chair: William Ross	**Session B (Room 125)** Chair: Dijana Ilišević
14:30 – 14:50	**Dariusz Cichoń** Uniqueness of a positive extension in the complex moment problem and a representing measure	**Jacek Chmieliński** On operators reversing Birkhoff orthogonality
14:55 – 15:15	**Gökhan Göğüş** Composition operators on Dirichlet-type and Möbius invariant spaces	**Michał Wojtylak** The Crouzeix conjecture and deformations of the numerical range
15:20 – 15:40	**Bartosz Łanucha** On asymmetric truncated Toeplitz operators of rank one	**Alan Kamuda** Towards Theory of Frames in Krein Spaces
15:40 – 16:00	**Coffee break (Room 127)**	
	Session A (Room 120) Chair: Petru Cojuhari	**Session B (Room 125)** Chair: Lina Oliveira
16:00 – 16:20	**Marcin Moszyński** Spectrum of block-Jacobi operator – how to adopt some scalar Jacobi operator methods?	**Adam Wegert** Canonical commutation relation, traces and affiliated operators
16:25 – 16:45	**Ewelina Zalot** Spectral resolutions for non-self-adjoint block Jacobi matrices	**Marlena Nowaczyk** Scattering on the line with a transfer condition at the origin
16:50 – 17:10	**Marcin Zygmunt** Spectral Analysis of Block Jacobi Matrices with Singular Entries	**Paweł Sobolewski** On balayage and B-balayage

Tuesday, July 10th

Conference Hall: al. Mickiewicza 24/28

Plenary session (Room 120) Chair: Cristina Câmara	
9:00 – 9:40	**Alexei Poltoratski** Completeness problems for complex exponentials
9:45 – 10:25	**Eungil Ko** On square roots of self-adjoint weighted composition operators on H^2
10:25 – 10:55	**Coffee break (Room 127)**
Chair: Jan Stochel	
10:55 – 11:25	**Piotr Budzyński** Weighted composition operators in L^2-spaces
11:30 – 12:00	**Lina Oliveira** Invariant subspace lattices and kernel maps
11:50 – 14:30	**Lunch**

Tuesday July 10th

Conference Hall: al. Mickiewicza 24/28

Parallel sessions		
	Session A (Room 120) Chair: Albrecht Böttcher	**Session B (Room 125)** Chair: Dan Timotin
14:30 – 14:50	**Maria Nowak** Examples of de Branges-Rovnyak spaces generated by nonextreme functions	**Ekaterina Shulman** Approximation of invariant subsets by invariant subspaces with application to Levi-Civita functional equations
14:55 – 15:15	**Bruce Watson** The one-dimensional p-Laplacian with indefinite weight	**Małgorzata Michalska** Matrix representations of truncated Hankel operators
15:20 – 15:40	**Sourav Pal** A Nagy-Foias program for the $C_{\cdot 0}$ operator tuples associated with the symmetrized polydisc	**Joanna Jurasik** Asymmetric truncated Toeplitz operators on finite-dimensional spaces
15:40 – 16:00	**Coffee break (Room 127)**	
	Session A (Room 120) Chair: Jacek Chmieliński	**Session B (Room 125)** Chair: Bruce Watson
16:00 – 16:20	**Tamás Titkos** Generalized Krein-von Neumann extension	**Jani Virtanen** Toeplitz operators on Hardy spaces with piecewise continuous symbols
16:25 – 16:45	**Zsigmond Tarcsay** Generalized Schur complementation	**Roksana Słowik** Doubly infinite matrices and the Cayley-Hamiltonian theorem

Wednesday, July 11th

Conference Hall: al. Mickiewicza 24/28

Plenary session (Room 120) Chair: Javad Mashreghi	
9:00 – 9:40	**Man Duen-Choi** A Mathematician's Apology on Tensor Products
9:45 – 10:25	**Zenon Jabłoński** Cauchy Dual operators, kernel condition and quasi-Brownian isometries
10:25 – 10:55	**Coffee break (Room 127)**
Chair: Roman Drnovšek	
10:55 – 11:25	**Albrecht Böttcher** On the norm of the Laplace operator on spaces of polynomials
11:30 – 12:00	**Petru Cojuhari** Spectral theory for perturbed block Toeplitz operators
12:00 – 14:30	**Lunch**

Wednesday, July 11th

Conference Hall: al. Mickiewicza 24/28

	Session A (Room 120) Chair: Andrzej Sołtysiak	**Session B (Room 125)** Chair: Maria Nowak
14:30 – 14:50	**Krzysztof Rudol** Gleason parts in bidual algebras	**Wen-Chi Kuo** Martingales, mixingales and mixing processes in Riesz spaces
14:55 – 15:15	**Ja A Jeong** The ideal structure of labeled graph C^*-algebras	**Renata Rososzczuk** Extremal problem for invariant subspaces in the weighted Bergman spaces A^p_α, $\alpha > -1$, $p > 0$.
15:20 – 15:40	**Jakub Kośmider** Unitary equivalence of weighted shifts	**Volodymyr Dilnyi** Weighted Hardy spaces and signal processing
15:40 – 16:00	**Coffee break (Room 127)**	
	Session A (Room 120) Chair: Zenon Jabłoński	**Session B (Room 125)** Chair: Eungil Ko
16:00 – 16:20	**Patryk Pagacz** Invariant subspaces of $H^2(\mathbb{T}^2)$ and $L^2(\mathbb{T}^2)$ preserving compatibility	**Shubhankar Podder** Commutants and reflexivity of multiplication tuples on vector-valued reproducing kernel Hilbert spaces
16:25 – 16:45	**Artur Płaneta** Generalized multipliers for left-invertible analytic operators	**Paweł Pietrzycki** Jensen's inequality for operators and commutativity
16:50 – 17:10		**Elroy Zeekoei** tba

Thursday, July 12th

Conference Hall: al. Mickiewicza 24/28

Plenary session (Room 120) Chair: Alexei Poltoratski	
9:00 – 9:40	**Javad Mashreghi** The Gleason-Kahane-Zelazko theorem in function spaces
9:45 – 10:25	**Vladimír Müller** Diagonals of operators and pinching
10:25 – 10:55	**Coffee break (Room 127)**
11:15- 18.00	**Excursion**
19:00- 22:00	**Conference dinner**

Friday, July 13th

Conference Hall: al. Mickiewicza 24/28

Parallel sessions		
	Session A (Room 120) Chair: Kamila Kliś	**Session B (Room 125)** Chair: Victor Kaftal
9:00 – 9:20	**Lech Zielinski** On the spectrum of the quantum Rabi model	**Roman Drnovšek** Triangularizability of trace-class operators with increasing spectrum
Plenary session (Room 120) Chair: Man Duen-Choi		
9:25 – 10:05	**Victor Kaftal** Diagonals of positive operators, an alternative approach	
10:10 – 10:50	**Cristina Câmara** Completions of partial operator matrices	
10:50 –	**Last coffee (Room 127)**	

List of Participants

Beberok, Tomasz
University of Agriculture in Krakow, Poland
e-mail: tomasz.beberok@urk.edu.pl

Białas-Cieź, Leokadia
Jagiellonian University in Kraków, Poland
e-mail: leokadia.bialas-ciez@uj.edu.pl

Böttcher, Albrecht
TU Chemnitz, Germany
e-mail: aboettch@math.tu-chemnitz.de

Budzyński, Piotr
University of Agriculture in Krakow, Poland
e-mail: piotr.budzynski@urk.edu.pl, piotr.budzynski@outlook.com

Burdak, Zbigniew
University of Agriculture in Krakow, Poland
e-mail: z.burdak@ur.krakow.pl

Câmara, M. Cristina
Instituto Superior Técnico-Universidade de Lisboa, Portugal
e-mail: ccamara@math.tecnico.ulisboa.pt

Chmieliński, Jacek
Pedagogical University of Cracow, Poland
e-mail: jacek.chmielinski@up.krakow.pl

Choi, Man-Duen
University of Toronto, Canada
e-mail: choi@math.toronto.edu

Cichoń, Dariusz
Jagiellonian University in Kraków, Poland
e-mail: dariusz.cichon@uj.edu.pl

Cojuhari, Petru
AGH University of Science and Technology, Poland
e-mail: cojuhari@agh.edu.pl

Das, Laxminarayan
Delhi Technological University, India
e-mail: lndas@dce.ac.in

Dębowska, Kamila
AGH University of Science and Technology, Poland
e-mail: kdebowska@agh.edu.pl

Dilnyi, Volodymyr
Cracow University of Technology, Poland
e-mail: dilnyiv@gmail.com

Drnovšek, Roman
University of Ljubljana, Slovenia
e-mail: roman.drnovsek@fmf.uni-lj.si

Dymek, Piotr
University of Agriculture in Kraków, Poland
e-mail: piotr.dymek@urk.edu.pl

Göğüş, Gökhan
Sabanci University, Turkey
e-mail: nggogus@sabanciuniv.edu

González, Manuel
Universidad de Cantabria, Spain
e-mail: manuel.gonzalez@unican.es

Hryniv, Rostyslav
University of Rzeszów, Poland
e-mail: rhryniv@gmail.com

Ilišević, Dijana
University of Zagreb, Croatia
e-mail: ilisevic@math.hr

Jabloński, Zenon
Jagiellonian University in Kraków, Poland
e-mail: Zenon.Jablonski@im.uj.edu.pl

Jeong, Ja A
Seoul National University, Republic of Korea
e-mail: jajeong@snu.ac.kr

Jurasik, Joanna
Maria Curie-Sklodowska University, Poland
e-mail: asia.blicharz@op.pl

Kaftal, Victor
University of Cincinnati, USA
e-mail: Kaftalv@ucmail.uc.edu

Kamuda, Alan
AGH University of Science and Technology, Poland
e-mail: alan.kamuda@gmail.com

Kliś, Kamila
University of Agriculture in Kraków, Poland
e-mail: rmklis@cyfronet.pl

Ko, Eungil
Ewha Womans University, Korea
e-mail: eiko@ewha.ac.kr

Kosiek, Marek
e-mail: Marek.Kosiek@im.uj.edu.pl

Kośmider, Jakub
Jagiellonian University in Kraków, Poland
e-mail: jakub.kosmider@im.uj.edu.pl

Kowalska, Agnieszka
Pedagogical University of Cracow, Poland
e-mail: agnieszka.kowalska@up.krakow.pl

Kuo, Wen-Chi
University of the Witwatersrand, South Africa
e-mail: wcakuo@gmail.com

Łanucha, Bartosz
Maria Curie-Sklodowska University, Lublin, Poland
e-mail: bartosz.lanucha@poczta.umcs.lublin.pl

Mashreghi, Javad
Laval University, Canada
e-mail: javad.mashreghi@mat.ulaval.ca

Michalska, Małgorzata
Maria Curie-Sklodowska University, Lublin, Poland
e-mail: malgorzata.michalska@poczta.umcs.lublin.pl

Moszyński, Marcin
University of Warsaw, Poland
e-mail: mmoszyns@mimuw.edu.pl

Müller, Vladimír
Mathematical Institute, Czech Academy of Sciences, Czech Republic
e-mail: muller@math.cas.cz

Nowaczyk, Marlena
AGH University of Science and Technology, Poland
e-mail: marlena17nowaczyk@gmail.com

Nowak, Maria
Maria Curie-Sklodowska University, Lublin, Poland
e-mail: mt.nowak@poczta.umcs.lublin.pl

Oliveira, Lina
Instituto Superior Técnico - Universidade de Lisboa, Portugal
e-mail: linaoliv@math.tecnico.ilisboa.pt

Pagacz, Patryk
Jagiellonian University in Kraków, Poland
e-mail: patryk.pagacz@gmail.com

Pal, Sourav
Indian Institute of Technology Bombay, India
e-mail: sourav@math.iitb.ac.in

Park, Gi Hyun
Hanshin University, Republic of Korea
e-mail: ghpark@hs.ac.kr

Pietrzycki, Paweł
Jagiellonian University in Kraków, Poland
e-mail: pawel.pietrzycki@im.uj.edu.pl

Płaneta, Artur
University of Agriculture in Kraków, Poland
e-mail: artur.planeta@urk.edu.pl

Podder, Shubhankar
Harish-Chandra Research Institute, India
e-mail: shubhankarpodder@hri.res.in

Poltoratski, Alexei
Texas A&M University, USA
e-mail: apoltora@gmail.com

Ptak, Marek
University of Agriculture in Krakow, Poland
e-mail: rmptak@cyf-kr.edu.pl

Rososzczuk, Renata
Lublin University of Technology, Poland
e-mail: r.rososzczuk@pollub.pl

Ross, William
University of Richmond, USA
e-mail: wross@richmond.edu

Rudol, Krzysztof
AGH University of Science and Technology, Polska
e-mail: rudol@agh.edu.pl

Shulman, Ekaterina
University of Silesia in Katowice, Poland
e-mail: ekaterina.shulman@us.edu.pl

Słowik, Roksana
Silesian University of Technology, Poland
e-mail: roksana.slowik@gmail.com

Sobolewski, Paweł
Maria Curie-Sklodowska University, Poland
e-mail: ptsob@hektor.umcs.lublin.pl

Sołtysiak, Andrzej
Adam Mickiewicz University, Poznań, Poland
e-mail: asoltys@amu.edu.pl

Sroka, Grzegorz
Politechnika Rzeszowska, Poland
e-mail: gsroka@prz.edu.pl

Stochel, Jan
Jagiellonian University in Kraków, Poland
e-mail: jan.stochel@im.uj.edu.pl

Tarcsay, Zsigmond
Eötvös Loránd University, Budapest, Hungary
e-mail: tarcsay@cs.elte.hu

Timotin, Dan
Institute of Mathematics Simion Stoilow of the Romanian Academy, Romania
e-mail: dan.timotin@gmail.com

Titkos, Támas
Rényi Institute, Hungary
e-mail: titkos.tamas@renyi.mta.hu

Virtanen, Jani
University of Reading, England
e-mail: j.a.virtanen@reading.ac.uk

Watson, Bruce
University of the Witwatersrand, South Africa
e-mail: b.alastair.watson@gmail.com

Wegert, Adam
AGH University of Science and Technology, Poland
e-mail: a_wegert@o2.pl

Wojtylak, Michał
Jagiellonian University in Kraków, Poland
e-mail: michal.wojtylak@gmail.com

Zalot, Ewelina
AGH University of Science and Technology, Polska
e-mail: zalot@agh.edu.pl

Zeekoei, Elroy
North-West University, South Africa
e-mail: elroy.zeekoei@nwu.ac.za

Zielinski, Lech
Université du Littoral Côte d'Opale, France
e-mail: lech.zielinski@lmpa.univ-littoral.fr

Zygmunt, Marcin
AGH University of Science and Technology, Poland
e-mail: zygmunt@agh.edu.pl

Abstracts

On the norm of the Laplace operator on spaces of polynomials

Albrecht Böttcher

In joint work with Peter Dörfler, we derived Markov-type inequalities for certain partial differential operators on multivariate polynomials with Hilbert space norms. Unfortunately, these results are not applicable to the most important partial differential operator, the Laplace operator. The talk is devoted to first insights gained for just this operator.

We consider the Laplace operator on the finite-dimensional linear space of algebraic polynomials in k variables such that each variable occurs at most with the power n. The space of the polynomials is equipped with the Laguerre norm. We establish safe lower and upper bounds for the norm of the Laplace operator on this space, and we derive asymptotic lower and upper bounds for this norm as n goes to infinity. The asymptotic bounds are better than the safe bounds.

The talk is based on joint work with Christian Rebs.

Weighted composition operators in L^2-spaces

Piotr Budzyński

Given a σ-finite measure space $(X, \mathscr{A}, \mu)$, an $\mathscr{A}$-measurable transformation ϕ of X, and a complex $\mathscr{A}$-measurable function w on X, the weighted composition operator in $L^2(\mu)$ induced by ϕ and w is given by

$$\mathcal{D}(C_{\phi,w}) = \{f \in L^2(\mu) \colon w \cdot (f \circ \phi) \in L^2(\mu)\},$$
$$C_{\phi,w} f = w \cdot (f \circ \phi), \quad f \in \mathcal{D}(C_{\phi,w}),$$

The talk is aimed at presenting a variety of results concerning normality, seminormality, selfadjointness, and subnormality of bounded and unbounded weighted composition operators acting in L^2-spaces.

The talk is based on joint work with Zenon Jabłoński, Il Bong Jung, and Jan Stochel.

[1] P. Budzyński, Z. J. Jabłoński, I. B. Jung, J. Stochel, *Unbounded weighted composition operators in L^2-spaces*, Lecture Notes in Math. 2209 (2018).

Completions of partial operator matrices

M. Cristina Câmara

We consider complex symmetric completions of a partial operator matrix whose specified part is an operator from a Hilbert space H into a closed proper subspace. We give necessary and sufficient conditions for such a completion to exist, with respect to a given conjugation C, and we describe all possible completions of that type. We apply these results to completion problems for various classes of partial operator matrices, in particular partial rectangular Toeplitz matrices and block Toeplitz operators.

The talk is based on joint work with Kamila Kliś-Garlicka and Marek Ptak.

On operators reversing Birkhoff orthogonality

Jacek Chmieliński

Let $(X, \|\cdot\|)$ be a normed space over the scalar field $\mathbb{K}$ and let $\perp_{\mathrm{B}}$ denote the Birkhoff orthogonality, i.e.,

$$x \perp_{\mathrm{B}} y \iff \forall\, \lambda \in \mathbb{K}: \ \|x + \lambda y\| \geq \|x\| \qquad (x, y \in X).$$

A linear operator $T\colon X \to X$ *reverses orthogonality* (cf. [1]) iff

$$x \perp_{\mathrm{B}} y \implies Ty \perp_{\mathrm{B}} Tx, \qquad x, y \in X.$$

As opposed to the planar case, if $\dim X \geq 3$, the existence of such operators characterizes inner product spaces.

We consider also operators which *approximately* reverse orthogonality. They may exist also in higher dimensional normed spaces which are not inner product ones.

The talk partially refers to a joint work with Paweł Wójcik.

[1] J. Chmieliński: Operators reversing orthogonality in normed spaces, *Adv. Oper. Theory*, 1 (2016), 8–14.

A Mathematician's Apology on Tensor Products

Man-Duen Choi

This is an expository lecture on the structure of tensor products of complex matrices. In all times of my mathematical journey, I have beautiful dreams of non-commutative geometry. Suddenly, I was awaken in the new era of Quantized world with fantasies and controversies. To release myself from Quantum Entanglements and the Principle of Locality, I need to seek the meaning of physics and the value of metaphysics. Conclusion: I THINK, THEREFORE I AM a pure mathematician.

Uniqueness of a positive extension in the complex moment problem and a representing measure

Dariusz Cichoń

A well known characterization of complex moment sequences on Sz.-Nagy semigroup $\mathbb{N} \times \mathbb{N}$ (i.e. sequences admitting a representing measure) consist in requiring existence of a positive definite extensions on a larger semigroup of pairs of integers (m, n) with $m + n$ nonnegative. In this talk we consider the question of determinacy for complex moment sequences with the connection to uniqueness of its positive definite extensions. This uniqueness is tightly related to the condition on the representing measures of the complex moment sequence having no atom at the point 0, which is in contrast with indeterminate Hamburger moment sequences (as they always admit a representing measure with atom at 0). Considering examples of indeterminate sequences we employ representing measures supported on algebraic sets, which among others will be covered in the talk. The content of the talk is based on the joint work with J. Stochel and F.H. Szafraniec.

Spectral theory for perturbed block Toeplitz operators

Petru Cojuhari

We propose to discuss spectral properties, mainly important for scattering theory, of certain perturbed Toeplitz operators. Problems are treated for the general case, in an abstract framework, using direct methods of perturbation theory. Applications to block Jacobi matrices and also to perturbed periodic Jacobi matrices will be considered. In particular, estimates for the number of the eigenvalues created by perturbations in the spectral gaps will be also presented.

Weighted Hardy spaces and signal processing

Volodymyr Dilnyi

Let $H^p_\sigma(\mathbb{C}_+)$, $\sigma \geq 0$, be a space of holomorphic in $\mathbb{C}_+$ functions such that

$$\|f\| := \sup_{\varphi\in(-\frac{\pi}{2};\frac{\pi}{2})} \left\{ \int_0^{+\infty} |f(re^{i\varphi})|^p e^{-p\sigma r|\sin\varphi|} dr \right\}^{1/p} < +\infty.$$

We denote a half-strip $\{z : |\mathrm{Im}\, z| < \sigma, \mathrm{Re}\, z < 0\}$ by D_σ. The *filter identification problem* for the half-strip D_σ is to find, if possible, a test signal g belonging to the Hardy space E^2 in $D^*_\sigma = \mathbb{C}\backslash\overline{D}_\sigma$ whose output

$$g * f(\tau) = \int_{\partial D_\sigma} g(w) f(w+\tau) dw$$

measured at all time moments $\tau \leq 0$ defines uniquely an unknown filter $f \in E^2[D_\sigma]$. More precisely, the question is whether there exists $g \in E^2_*[D_\sigma]$ such that $g * f(\tau) = 0$ for all $\tau \leq 0$ implies $f \equiv 0$?

The amplitude spectrum of $g \in E^2[D^*_\sigma]$ is defined by the equality

$$g(w) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} G(x) e^{-xw} dx, \qquad \mathrm{Re}\, w > 0$$

and belongs to the $H^2_\sigma(\mathbb{C}_+)$.

Theorem 1. *Suppose the amplitude spectrum G of a signal $g \in E^2[D^*_\sigma]$ is continuous and zero-free in $\{z : \Re z \geq 0\}$, $f \in E^2[D_\sigma]$. Then $f * g(\tau) = 0$ for all $\tau \leq 0$ implies $f \equiv 0$ if and only if one of the following conditions holds:*
1) g admits a holomorphic continuation as an entire function and

$$(\forall c \in \mathbb{R}) : g(w) \exp\left(-ce^{-\frac{w\pi}{2\sigma}}\right) \notin E^2[D_\sigma];$$

2) g does not admit an analytic continuation to an entire function.

The talk is based on joint work with Khrystyna Huk.

[1] Nikolski N.K., *Operatos, functions and systems: an easy reading.* V.1-2, AMS, 2002.

Triangularizability of trace-class operators with increasing spectrum

Roman Drnovšek

For any measurable set E of a measure space (X,μ), let P_E be the (orthogonal) projection on the Hilbert space $L^2(X,\mu)$ with the range

$$\operatorname{ran} P_E = \{f \in L^2(X,\mu) : f = 0 \text{ a.e. on } E^c\}$$

that is called a standard subspace of $L^2(X,\mu)$. Let T be an operator on $L^2(X,\mu)$ having increasing spectrum relative to standard compressions, that is, for any measurable sets E and F with $E \subseteq F$, the spectrum of the operator $P_E T|_{\operatorname{ran} P_E}$ is contained in the spectrum of the operator $P_F T|_{\operatorname{ran} P_F}$. The authors of [1] asked whether the operator T has a non-trivial invariant standard subspace. They answered this question affirmatively when either the measure space (X,μ) is discrete or the operator T has finite rank. We study this problem in the case of trace-class kernel operators.

[1] L. W. Marcoux, M. Mastnak, H. Radjavi: Triangularizability of operators with increasing spectrum, *J. Funct. Anal.*, 257 (2009), 3517–3540.

[2] R. Drnovšek: Triangularizability of trace-class operators with increasing spectrum, *J. Math. Anal. Appl.*, 447 (2017), 1102–1115.

Composition operators on Dirichlet-type and Möbius invariant spaces

Gökhan Göğüş

In this talk, norm and essential norm estimates of composition operators acting on Dirichlet-type spaces and Möbius invariant spaces of analytic functions on the unit disk will be presented. The Dirichlet-type spaces were introduced recently for the first time by G. Bao, G. Gogus and S. Pouliasis.

The work related to this talk is mostly done jointly with Guanlong Bao and Stamatis Pouliasis.

On isometric roots of isometries

Dijana Ilišević

Any power of an isometry is again an isometry. What about the converse of this fact? More precisely, is it true that a given isometry is a power of some isometry (with respect to the same norm)? In other words, for a given positive integer k, does there exist an isometric kth root of a given isometry? The aim of this talk is to answer this question in the setting of various normed spaces. It will be shown that the answer depends on a given norm and a given power k, it differs in real and in complex spaces, and it also differs in finite and in infinite dimensions. In particular, examples of isometries with and without isometric roots will be presented.

The talk is based on joint work with Bojan Kuzma from the University of Primorska, Koper, Slovenia.

The work of Dijana Ilišević has been fully supported by the Croatian Science Foundation under the project IP-2016-06-1046.

Cauchy Dual operators, kernel condition and quasi-Brownian isometries

Zenon Jabłoński

Let T be a bounded linear operator on a complex Hilbert space $\mathcal{H}$. For a left invertible operator T, the Cauchy dual T' of T is given by $T' = T(T^*T)^{-1}$. W say that T is a 2-isometry, if $I - 2T^*T + T^{*2}T^2 = 0$. An operator T is called a quasi-Brownian isometry if T is a 2-isometry such that $\triangle_T T = \triangle_T^{1/2} T \triangle_T^{1/2}$, where $\triangle_T = T^*T - I$. Finally, we say that an operator T satisfies the kernel condition if $T^*T(\ker T^*) \subseteq \ker T^*$. In the talk we discuss several interesting properties of the above mentioned operators.

The talk is based on joint work with A. Anand, S. Chavan and J. Stochel.

The ideal structure of labeled graph C^*-algebras

Ja A Jeong

Associated to labeled graphs $(E, \mathcal{L})$ a class of C^*-algebras $C^*(E, \mathcal{L})$, called labeled graph C^*-algebras, was introduced by Bates and Pask in [1] and studied in the papers [2, 3] sequel to [1]. A C^*-algebra in this class is defined to be a C^*-algebra generated by a universal family of partial isometries satisfying certain relations determined by the labeled graph in question. By the universal property, a labeled graph C^*-algebra always carries a gauge action of the unit cicle. We will discuss the gauge-invariant ideal structure of labeled graph C^*-algebras based on the joint works ([5], [6]) with S. H. Kim and G. Park.

[1] T. Bates and D. Pask: C^*-algebras of labelled graph, *J. Operator Theory*, 57:1 (2007), 207–226.

[2] T. Bates and D. Pask: C^*-algebras of labelled graph II - Simplicity Results, *Math. Scand.*, 104 (2009), 249–274.

[3] T. Bates, T. M. Carlsen, and D. Pask: C^*-algebras of labelled graph III - K-theory computations, *Ergod. Th. & Dynam. Sys.*, 37 (2017), 337–368.

[5] J. A Jeong, S. H. Kim and G. H. Park: The structure of gauge-invariant ideals of labelled graph C^*-algebras, *J. Funct. Anal.*, 262 (2012), 1759–1780.

[6] J. A Jeong and G. H. Park: Simple labeled graph C^*-algebras are associated to disagreeable labeled spaces, *J. Math. Anal. App.*, 461 (2018), 1391–1403.

Asymmetric truncated Toeplitz operators on finite-dimensional spaces

Joanna Jurasik

Let H^2 be the usual Hardy space, the subspace of L^2 of normalized Lebesgue measure on $\mathbb{T}$ whose negative indexed Fourier coefficients are all zero. With any nonconstant inner function α we associate the model space K_α, defined by $K_\alpha = H^2 \ominus \alpha H^2$. Truncated Toeplitz operators are compressions of classical Toeplitz operators to model spaces. We consider their generalizations, the so-called asymmetric truncated Toeplitz operators. Let α, β be two inner functions and let

$\varphi \in L^2$. An asymmetric truncated Toeplitz operator $A_\varphi^{\alpha,\beta}$ is the operator from K_α into K_β given by

$$A_\varphi^{\alpha,\beta} f = P_\beta(\varphi f), \quad f \in K_\alpha,$$

where P_β is the orthogonal projection of L^2 onto K_β.

We present some known properties of asymmetric truncated Toeplitz operators. In particular, we present their characterizations in terms of matrix representations and compare with the characterizations of truncated Toeplitz operators.

The talk is based on joint work with Bartosz Łanucha.

[1] C. Câmara, K. Kliś-Garlicka, J. Jurasik, M. Ptak: Characterizations of asymmetric truncated Toeplitz operators, *Banach J. Math. Anal.*, 11 (2017), no. 4, 899–922.

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[3] J. Jurasik, B. Łanucha: Asymmetric truncated Toeplitz operators equal to the zero operator, *Ann. Univ. Mariae Curie-Skłodowska Sect. A* 70 (2016), no. 2, 51–62.

[4] J. Jurasik, B. Łanucha: Asymmetric truncated Toeplitz operators on finite-dimensional spaces, *Oper. Matrices*, 11 (2017), no. 1, 245–262.

[5] D. Sarason: Algebraic properties of truncated Toeplitz operators, *Operators and Matrices*, 1 (2007), no. 4, 491–526.

Diagonals of positive operators, an alternative approach

Victor Kaftal

I will survey the problem of finding the diagonals $\operatorname{diag}\xi$ of a positive operator A, from the classic Schur-Horn theorem to the more recent Kadison Pythagorean theorem.

Recognizing that this problem is equivalent to finding the coefficients in the decomposition of $A = \sum_j \xi_j P_j$ where P_j are rank-one projections, provides an additional, sometimes easier, approach.

An illustration of this fact is the new proof of the sufficiency condition in the Kadison Pythagorean theorem, not only for the case that A is a projection, but also when A is a sum of (not necessarily mutually orthogonal) projections.

The talk is based on joint work with David Larson.

[1] R. Kadison: The Pythagorean Theorem II: the infinite discrete case, *Proc. Natl. Acad. Sci. USA*, 99 (2002), 5217–5222.

[2] V. Kaftal, J. Loreau: Kadison's Pythagorean Theorem and essential codimension, *Int Eq. Oper. Th,* 87 (2017), 565–580.

[3] V. Kaftal, D. Larson: Admissible sequences of positive operators. To appear *Trans AMS.*

Towards Theory of Frames in Krein Spaces

Alan Kamuda

During the first part of the talk we discuss the difference between two contemporary definitions of frames in Krein spaces. The first definition, was introduced in 2012 by Julian I. Giribet et al.

[1] and called J-frame. The second one was defined in 2015 by Kevin Esmeral et al. [2]. We briefly mention conclusions related to these definitions and properties of associated frame operators. In the second part, according to [3], we introduce a new definition of frames in Krein spaces which generalizes the previous ones and fits well the ideology of Krein spaces. In the last part we present the methodology of J-frame construction with using the conventional frame from Hilbert space and we present two examples of J-frames.

The talk is based on the following papers.

[1] J. I. Giribet, A. Maestripieri, F. Martinez Peria, P. G. Massey: On frames for Krein spaces, *J. Math. Anal. Appl.*, 393 (2012).

[2] K. Esmeral, O. Ferrer, E. Wagner: Frames in Krein spaces arising from a non-regular W-metric, *Banach J. Math. Anal.*, 9:1, (2015) 1–16.

[3] S. Kużel, A. Kamuda: On J-frames related to maximal definite subspaces, *Annals of Functional Analysis*, accepted for publication.

On square roots of self-adjoint weighted composition operators on H^2

Eungil Ko

In this talk, we characterize square roots of self-adjoint weighted composition operators on the Hardy space H^2, i.e., $W_{g,\psi}$ such that $W_{g,\psi} = W_{f,\varphi}^2$ is self-adjoint. In particular, we find symbol functions f and φ in this case. Some of $W_{f,\varphi}$ may be other, nonself-adjoint weighted composition operators. We also investigate several properties of such $W_{f,\varphi}$. Finally, we give equivalent conditions for such $W_{f,\varphi}$ to be self-adjoint or normal, respectively.

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2016R1D1A1B03931937).

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[2] C. C. Cowen: The commutants of an analytic Toeplitz operator, *Trans. Amer. Math. Soc.*, 239 (1978), 1-31.

[3] C. C. Cowen: An analytic Toeplitz operator that commutes with a compact operator, *J. Functional Analysis*, 36(2) (1980), 169-184.

[4] C. C. Cowen: Composition operators on H^2, *J. Operator Theory*, 9 (1983), 77-106.

[5] C. C. Cowen: Linear fractional composition operators on H^2, *Int. Eq. Op. Th.*, 11 (1988), 151-160.

[6] C. C. Cowen, S. Jung, and E. Ko: Normal and cohyponormal weighted composition operators on H^2, *Operator Theory: Adv. and Appl.*, 240 (2014), 69-85.

[7] C. C. Cowen and E. Ko: Hermitian weighted composition operators on H^2, *Trans. Amer. Math. Soc.*, 362 (2010), 5771-5801.

[8] C. C. Cowen and B. D. MacCluer: *Composition operators on spaces of analytic Functions*, CRC Press, Boca Raton, 1995.

[9] J. A. Deddens: Analytic Toeplitz and composition operators, *Canad. J. Math.*, 24 (1972), 859-865.

[10] P. Duren and A. Schuster: *Bergman Spaces,* Amer. Math. Soc., Providence, 2004.

[11] F. Forelli: The isometries of H^p, *Canad. J. Math.*, 16 (1964), 721-728.

[12] R. E. Greene and S. G. Krantz: Function theory of one complex variable, *Grad. Studies* **40**, *Amer. Math. Soc.*, 2002.

[13] H. Hedenmalm, B. Korenblum, and K. Zhu: *Theory of Bergman Spaces*, Springer-Verlag, New York, 2000.

[14] S. Jung, Y. Kim, and E. Ko: Characterizations of square roots of self-adjoint weighted composition operators on H^2, preprint.

[15] H. Radjavi and P. Rosenthal: *Invariant subspaces*, Springer-Verlag, 1973.

Unitary equivalence of weighted shifts

Jakub Kośmider

Let $\mathcal{H}$ be a nonzero Hilbert space and $\boldsymbol{B}(\mathcal{H})$ be the algebra of bounded operators defined on $\mathcal{H}$. Let $\{S_n\}_{n\in\mathbb{Z}} \subseteq \boldsymbol{B}(\mathcal{H})$ be a two-sided sequence of bounded nonzero operators such that $\{||S_n||\}_{n\in\mathbb{Z}}$ is bounded. We say that an operator $S\colon \oplus_{n\in\mathbb{Z}}\mathcal{H} \to \oplus_{n\in\mathbb{Z}}\mathcal{H}$ is a *bilateral operator valued weighted shift* defined on $\mathcal{H}$ if for all $x \in \oplus_{n\in\mathbb{Z}}\mathcal{H}$ it holds that

$$Sx = (\ldots, S_{-1}x_{-2}, \boxed{S_0x_{-1}}, S_1x_0, \ldots),$$

where $x = (\ldots, x_{-1}, \boxed{x_0}, x_1, \ldots)$ and $\boxed{x_0}$ denotes the central element of x.

This talk is based on my recent work related to unitary equivalence of bilateral operator valued weighted shifts.

Martingales, mixingales and mixing processes in Riesz spaces

Wen-Chi Kuo

Mixingales are stochastic processes which combine the concepts of martingales and mixing sequences. McLeish introduced the term mixingale at the 4th Conference of Stochastic Processes and Applications, at York University, Toronto in 1974. We generalize the concept of a mixingale to the measure-free Riesz space setting. This generalizes all of the $L_p; 1 \le p \le \infty$ variants. We also generalize the concept of uniform integrability to the Riesz space setting and prove that a weak law of large numbers holds for Riesz space mixingales. The connections between martingales, mixingales and mixing processes in both the L_p space and the Riesz space settings will be discussed.

This talk is based on joint work with Coenraad Labuschagne, Jessica Vardy and Bruce Watson.

On asymmetric truncated Toeplitz operators of rank one

Bartosz Łanucha

In this talk we describe all asymmetric truncated Toeplitz operators of rank one. We then compare our description to the one given by D. Sarason for rank-one truncated Toeplitz operators. We point out the cases when these descriptions differ.

The Gleason–Kahane–Żelazko theorem in function spaces

Javad Mashreghi

Let $T : H^p \to H^p$ be a linear mapping (no continuity assumption). What can we say about T if we assume that "it preserves outer functions"? Similarly, under what conditions does it preserve the inner functions? Another related question is to consider linear functionals $T : H^p \to \mathbb{C}$ (again, no continuity assumption) and ask about those functionals whose kernels do not include any outer function. We study such questions via an abstract result which can be interpreted as the generalized Gleason–Kahane–Żelazko theorem for modules. In particular, we see that continuity of endomorphisms and functionals is a part of the conclusion. We discuss similar questions in other function spaces, e.g., Bergman, Dirichlet, Besov, the little Bloch, and VMOA.

This is a joint work with T. Ransford.

Matrix representations of truncated Hankel operators

Małgorzata Michalska

Truncated Hankel operators are compressions of classical Hankel operators to model spaces. In this talk we describe truncated Hankel operators on finite-dimensional model spaces using their matrices with respect to some natural bases. We then extend our descriptions to some infinite-dimensional cases.

The talk is based on joint work with Bartosz Łanucha.

Spectrum of block-Jacobi operator – how to adopt some scalar Jacobi operator methods?

Marcin Moszyński

The Block Jacobi operator J is an analog of the classical scalar Jacobi operator. This is a self-adjoint operator in the Hilbert space $l^2(N, \mathbb{C}^d)$ (instead of $l^2(N, \mathbb{C})$). Block Jacobi operator J is determined by a tridiagonal matrix with terms ("diagonals" and "weights") being $d \times d$ "blocks" — d by d matrices (instead of scalar terms for the scalar Jacobi operator case). When $d > 1$ the spectral analysis of J makes much more difficult than in the scalar case $d = 1$, especially when we study unbounded block-weights.

During my talk I will explain some technical details of such problems. I will also formulate some conjectures and open questions.

Diagonals of operators and pinching

Vladimír Müller

Let T be a bounded linear operator on a separable Hilbert space H. We will discuss the question what are the possible diagonals $(\langle Te_k, e_k\rangle)$ of T with respect to an orthonormal basis (e_k) in H. The question will be also discussed for n-tuples of operators, especially for n-tuples of the form $(T, T^2, \dots, T^n)$ where $T \in B(H)$.

Similar techniques give also new results for the "pinching" of operators.

The talk is based on joint work with Yu. Tomilov.

Scattering on the line with a transfer condition at the origin

Marlena Nowaczyk

We consider the scattering problem of Sturm-Liouville operator on the line with a point transfer condition at the origin. The condition at the origin is described by transfer matrix M. We investigate both forward and inverse problems. In the forward problems the nature of the spectrum (finitely many, simple, negative eigenvalues) as well as conditions which characterise transfer conditions resulting in self-adjoint problem will be presented. For the inverse problems we will show that the transfer matrix can be reconstructed from the set of eigenvalues and the reflection coefficient.

Examples of de Branges-Rovnyak spaces generated by nonextreme functions

Maria Nowak

Let H^2 denote the standard Hardy space in the open unit disk $\mathbb{D}$ and let $\mathbb{T} = \partial\mathbb{D}$. For $\chi \in L^\infty(\mathbb{T})$ let T_χ denote the bounded Toeplitz operator on H^2, that is, $T_\chi f = P_+(\chi f)$, where P_+ is the orthogonal projection of $L^2(\mathbb{T})$ onto H^2. Given a function b in the unit ball of H^∞, the *de Branges-Rovnyak space* $\mathcal{H}(b)$ is the image of H^2 under the operator $(I - T_b T_{\bar{b}})^{1/2}$. The space $\mathcal{H}(b)$ is given the Hilbert space structure that makes the operator $(I - T_b T_{\bar{b}})^{1/2}$ a coisometry of H^2 onto $\mathcal{H}(b)$, namely

$$\langle (I - T_b T_{\bar{b}})^{1/2} f, (I - T_b T_{\bar{b}})^{1/2} g\rangle_b = \langle f, g\rangle_2 \quad (f, g \in (\ker(I - T_b T_{\bar{b}})^{1/2})^\perp).$$

Here we assume that b is a non-extreme point of the unit ball of H^∞. Then there exists a unique outer function $a \in H^\infty$ such that $a(0) > 0$ and $|a|^2 + |b|^2 = 1$ a. e. on $\mathbb{T}$. Then we say that (b, a) is a *pair*. If (b, a) is a pair, then the quotient $\varphi = b/a$ is in the Smirnov class $\mathcal{N}^+$. (Let us recall that the Smirnov class $\mathcal{N}^+$ consists of those holomorphic functions in $\mathbb{D}$ that are quotients of functions in H^∞ in which the denominators are outer functions.) Conversely, for every nonzero function $\varphi \in \mathcal{N}^+$ there exists a unique pair (b, a) such that $\varphi = b/a$ ([3]).

Many properties of $\mathcal{H}(b)$ can be expressed in terms of the function $\varphi = b/a$ in the Smirnov class $\mathcal{N}^+$. It is worth noting here that if φ is rational, then the functions a and b in the representation of φ are also rational (see [3]) and in such a case (b, a) is called a rational pair. Spaces $\mathcal{H}(b)$ for rational pairs have been studied, e.g., in [1] and [2].

Here we describe de Branges-Rovnyak spaces $\mathcal{H}(b_\alpha)$, $\alpha > 0$, where the corresponding Smirnov functions are $b_\alpha(z)/a_\alpha(z) = (1 - z)^{-\alpha}$, $\alpha > 0$

The talk is based on joint work with Bartosz Łanucha.

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[2] E. Fricain, A. Hartmann, W. T. Ross: Concrete examples of $\mathcal{H}(b)$ spaces, *Comput. Methods Func. Theory*, 16 (2016), no 2, 287–306.

[3] D. Sarason: Unbounded Toeplitz operators, *Integral Equations Operator Theory*, 61 (2008), 281-298.

Invariant subspace lattices and kernel maps

Lina Oliveira

We introduce the concept of the kernel map of an operator relative to a subspace lattice, outlining some of its properties and applications. In particular, we use these maps to show that any finite rank operator in a norm closed Lie module of a continuous nest algebra can be decomposed as a sum of finitely many rank-1 operators in the module. The hypothesis of the continuity of the nest cannot be dropped, in general.

The talk is based on joint work with Gabriel Matos.

Invariant subspaces of $\mathcal{H}^2(\mathbb{T}^2)$ and $L^2(\mathbb{T}^2)$ preserving compatibility

Patryk Pagacz

We consider the operators T_w, T_z of multiplication by independent variables "w", "z" on the space of square summable functions over the torus and its Hardy subspace. We say that an invariant subspace M preserves compatibility if a pair $(T_w|_M, T_z|_M)$ is compatible, i.e. $P_{T_w^n M} P_{T_z^m M} = P_{T_z^m M} P_{T_w^n M}$, for any $n, m \in \mathbb{N}$.

We describe an invariant subspaces of Hardy space $\mathcal{H}^2(\mathbb{T}^2)$ which preserves compatibility as ϕM_J, where ϕ is an inner function and M_J is a subspace $\bigvee_{(i,j)\in J} w^i z^j$, with some cone $J \subset \mathbb{N}^2$.

Moreover, we give a full description of invariant subspaces of $L^2(\mathbb{T}^2)$ preserving compatibility.

The talk is based on joint work with Zbigniew Burdak, Marek Kosiek and Marek Słociński.

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[2] Z. Burdak, M. Kosiek and M. Słociński: Compatible pairs of commuting isometries, *Linear Algebra Appl.*, 479 (2015), 216–259.

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A Nagy-Foias program for the $C_{\cdot 0}$ operator tuples associated with the symmetrized polydisc

Sourav Pal

A commuting tuple of operators $(S_1, \ldots, S_{n-1}, P)$, defined on a Hilbert space $\mathcal{H}$, for which the closed symmetrized polydisc

$$\Gamma_n = \left\{ \left(\sum_{1 \le i \le n} z_i, \sum_{1 \le i < j \le n} z_i z_j, \ldots, \prod_{i=1}^{n} z_i \right) : |z_i| \le 1, i = 1, \ldots, n \right\}$$

is a spectral set, is called a Γ_n-contraction. A Γ_n-contraction $(S_1, \ldots, S_{n-1}, P)$ is said to be $C_{\cdot 0}$ or *pure*, if $P^{*n} \to 0$ strongly as $n \to \infty$. We show an explicit construction of a Nagy-Foias type dilation and an operator model for the $C_{\cdot 0}$ Γ_n-contractions. Also we describe a complete unitary invariant for such operator tuples. This is an analogue of the Nagy-Foias complete unitary invariant for a contraction in terms of characteristic function.

Jensen's inequality for operators and commutativity

Paweł Pietrzycki

In 1973 M. R. Embry published a very influential paper studying the Halmos-Bram criterion for subnormality. In particular, she gave a characterization of the class of quasinormal operators in terms of powers of operators. Namely, bounded operator in Hilbert space is quasinormal if and only if the following condition holds

$$A^{*n} A^n = (A^* A)^n \qquad \text{for all} \quad n \in \mathbb{N}. \tag{1}$$

This leads to the following question: is it necessary to assume that the equality in (1) holds for all $n \in \mathbb{N}$? To be more precise we ask for which subset $S \subset \mathbb{N}$ the following system of operator equations:

$$A^{*s} A^s = (A^* A)^s \qquad \text{for all} \quad s \in S \tag{2}$$

implies the quasinormality of A.

We will prove that operator A is quasinormal if and only if it satisfies the system of equations (2) with $S = \{p, m, m+p, n, n+p\}$. This theorem generalizes Embry's characterization of quasinormality of bounded operators. We obtain a new characterization of the normal operators which resembles that for the quasinoraml operators.

[1] P. Pietrzycki: The single equality $A^{*n} A^n = (A^* A)^n$ does not imply the quasinormality of weighted shifts on rootless directed trees, *J. Math. Anal. Appl* 435 (2016), 338-348.

[2] P. Pietrzycki: Reduced commutativity of moduli of operators, *arXiv preprint arXiv:1802.01007*, 2018.

Generalized multipliers for left-invertible analytic operators

Artur Płaneta

A characterization of the commutant of a given operator is one of the way of investigation of the operator itself. The classical result on unilateral shift of (the multiplication by the independent variable on the Hardy space H^2) says that its commutant is the algebra of all multiplications by bounded analytic functions. In the case of unilateral shift of arbitrary multiplicity, its commutant is the algebra of all bounded analytic operator–valued functions. It was shown by Shields in [8], that the commutant of unilateral weighted shift of multiplicity one may be identified with the algebra of its multipliers. On the other hand, the multipliers for weighted shifts on rooted directed trees introduced in [1] are not sufficiently large to determine the whole commutant of the operator.

In this talk we present one possible approach to deal with the mentioned problem. As shown by Shimorin, every left-invertible analytic operator T on a Hilbert space is unitarily equivalent to a multiplication operator by z on a reproducing kernel Hilbert space of analytic functions on a disc with values in $\mathcal{N}(T^*)$ - the kernel of the adjoint of T. We define generalized multipliers for left-invertible analytic operator T using this unitary equivalence. This enable us to characterize the commutant of T. In particular, we discuss the application of general theory in the case of left-invertible weighted shifts on leafless rooted directed trees.

The talk is based on joint work with Piotr Dymek and Marek Ptak.

[1] P. Budzyński, P. Dymek, M. Ptak: Analytic structure of weighted shifts on directed trees, *Math. Nachr.*, 290 (2017), 1612–1629.

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Commutants and reflexivity of multiplication tuples on vector-valued reproducing kernel Hilbert spaces

Shubhankar Podder

In this talk, we address the question of identifying commutant and reflexivity of the multiplication d-tuple M_z on a reproducing kernel Hilbert space H of E-valued holomorphic functions on Ω, where E is a separable Hilbert space and Ω is a bounded domain in $\mathbb{C}^d$ admitting bounded approximation by polynomials. In case E is a finite dimensional cyclic subspace for M_z, under some natural conditions on the $B(E)$-valued kernel associated with H, the commutant of M_z is shown to be the algebra $H^\infty_{B(E)}(\Omega)$ of bounded holomorphic $B(E)$-valued functions on Ω, provided

M_z satisfies the matrix-valued von Neumann's inequality. Also, we show that a multiplication d-tuple M_z on H satisfying the von Neumann's inequality is reflexive.

The talk is based on joint work with Sameer Chavan and Shailesh Trivedi.

Completeness problems for complex exponentials

Alexei Poltoratski

I will talk about two well-known problems of harmonic analysis related to completeness of complex exponentials in L^2-spaces. The first problem was solved by Beurling and Malliavin in 1960s. I will discuss its modern generalizations, obtained via the use of Toeplitz operators, and further open problems. This part of the talk is based on joint work with Nikolai Makarov.

In the second part of the talk I will discuss the so-called type problem, which stood open until recently. Starting with a brief history of the problem, I will present recent results and new examples.

[1] N. Makarov and A. Poltoratski: Beurling-Malliavin Theory for Toeplitz Kernels, *Invent. Math.*, Vol. 180, Issue 3 (2010), 443–480.

[2] A. Poltoratski: A problem on completeness of exponentials, *Annals of Math.*, Volume 178 (2013), 983–1016

[3] A. Poltoratski: Toeplitz Approach to Problems of the Uncertainty Principle, Conference Board of the Mathematical Sciences (CBMS) series, AMS/NSF, 2015.

Extremal problem for invariant subspaces in the weighted Bergman spaces A^p_α, $\alpha > -1, p > 0$.

Renata Rososzczuk

We find an explicit formula for the solution of an extremal problem for the invariant subspaces in the weighted Bergman spaces $A^2_\alpha, -1 < \alpha < \infty$ consisting of functions having one zero of order at least k, $k = 0, 1, 1 \ldots$. A consequence of this formula is the failure of the Beurling-type theorem for $\alpha = 4$. We also consider the analogous extremal problem for $A^p_\alpha, 0 < p < \infty$.

The talk is based on joint work with M. T. Nowak, M. Wołoszkiewicz-Cyll.

Birkhoff-James Inner functions and zero sets of analytic functions

William Ross

In this joint work with Javad Mashreghi and Raymond Cheng, I will discuss the notion of an inner function for Hilbert and Banach space of analytic functions and how this notion can be used, as was used by Beurling to discuss the Hardy space setting, to describe the zero sets of functions from these spaces. There will be a particular focus on the space ℓ^p_A, the analytic functions whose power series coefficients belong to the sequence space ℓ^p.

Gleason parts in bidual algebras

Krzysztof Rudol

One of the methods of studying the Banach algebra $H^\infty(\Omega)$ is to view it as a subspace of the second dual A^{**} of the algebra $A = A(\Omega)$. The first algebra includes all bounded analytic functions on a domain $G \subset \mathbb{C}^d$, while $A(\Omega)$ consists of elements in $H^\infty(\Omega)$ having continuous extensions to $\overline{\Omega}$ and its spectrum is relatively easy to describe. The Arens multiplication in A^{**} extends the product in A under the natural embedding and even in the more general setup of function algebras, the bidual algebra is Arens-regular. Our study of the behaviour of Gleason parts and their closures under the passage to the second dual space initiated in [3] is continued, showing the delicate nature of the matter.

Unfortunately, there was a gap in [3], spotted by H.G. Dales [1] whose study of the bidual of $A = C(K)$ in [2] provided some hints in the cardinality of Gelfand closures of subsets of a compact space K. Consequently, most of the claims of Theorem 6 in [3] cannot hold in such generality. On the other hand, for the most interesting Gleason part related to the points of Ω we are able to obtain a number of positive results. Assuming some regularity, we show that the Gleason part in A^{**} that corresponds to the domain Ω is not containing any other points than the evaluation functionals at (the canonical image) of Ω. This can be even continued to higer-order duals of A.

Also some results on bands of measures representing points of Ω and the closures of their canonical embeddings in the bidual spaces will be presented, including an alternative (new) proof of F. and M. Riesz Theorem.

The talk is based on joint work with Marek Kosiek.

[1] H. G. Dales, Private communication (IX 2017).

[2] H. G. Dales, F. K. Dashiell, Jr., A. T.-M. Lau, and D. Strauss, *Banach Spaces of Continuous Functions as Dual Spaces*, Springer Verlag, New York, 2016.

[3] M. Kosiek, K. Rudol, Dual algebras and A – measures, *Journ. of Function Spaces*, 2014, 1–8 http://dx.doi.org/10.1155/2014/364271.

Approximation of invariant subsets by invariant subspaces with application to Levi-Civita functional equations

Ekaterina Shulman

Let T be a continuous representation of an amenable group G on a linear topological space X. Let $Y \subset X$ be a closed subspace of X invariant for T and supplied with a norm $\|.\|_Y$ such that the restriction of T to Y is an isometric representation. For $\xi \in X$ and any subspace $X_0 \subset X$, we define the Y-distance from ξ to X_0 by

$$d_Y(\xi, X_0) = \inf\{\|y\| : y \in Y,\ \xi - y \in X_0\}.$$

Statement: For each $\varepsilon > 0$ and $n \in \mathbb{N}$ there is a $\delta > 0$ with the following property:

if L is a finite-dimensional subspace of X with $\dim(L) \le n$ and

$$d_Y(T_g\xi, L) < \delta \qquad \text{for all } g \in G$$

then there is a T-invariant subspace $M \subset X$ with $\dim(M) \le 3n$ and

$$d_Y(\xi, M) < \varepsilon.$$

We apply the result to the study of stability of the Levi-Civita functional equation

$$f(gh) = \sum_{j=1}^{N} u_j(g)v_j(h), \qquad g, h \in G \tag{3}$$

in the class of measurable functions on an amenable group G. Stability in the Ulam-Hyers sense means that each function that "almost" satisfies the equation is "close" to a proper solution.

For bounded functions the study of stability of (3) leads to the following geometric problem: given a representation $g \mapsto T_g$ of a group G on a Banach space X, for any invariant subset $K \subset X$ to estimate its distance to invariant subspaces of X via the distances to arbitrary n-dimensional subspaces.

In this setting the problem was studied in [1] using a specially developed techniques of covariant width, and the estimate is more strict: $\dim(M) \leq \dim(L)$.

[1] E. Shulman,: Group representations and stability of functional equations, *J. London Math. Soc.*, 54 (1996), 111–120.

Doubly infinite matrices and the Cayley-Hamiltonian theorem

Roksana Słowik

Let K be a doubly infinite matrix. If K is a polynomial in S and S^{-1}, where by S we mean the shift matrix, then K is called a Laurent matrix. During the talk we will present some results connected to the following quenstion.

> If K is finite band and periodic (but not tridiagonal) is there a polynomial Q so that $Q(K)$ is a Laurent matrix?

We will show that the answer to the above question, in general, is no. We will also provide some examples and further directions to study this issue.

[1] M.I. Arenas-Herrera, L. Verde-Star: Representation of doubly infinite matrices as non-commutative Laurent series, *Spec. Matrices*, 5 (2017), 250-257.

[2] B. Simon: A Cayley-Hamilton theorem for periodic finite band matrices, *Functional analysis and operator theory for quantum physics*, EMS Ser. Congr. Rep., Eur. Math. Soc., Zürich, 2017, 525–529.

[3] R. Słowik: Some counterexamples for Cayley-Hamiltonian theorem for doubly infinite matrices, submitted.

On balayage and B-balayage

Paweł Sobolewski

Let $\mathbb{D}$ and $\mathbb{T}$ denote the open unit disk and the unit circle, respectively, in the complex plane $\mathbb{C}$. For a finite positive Borel measure μ on $\mathbb{D}$ the function

$$S_\mu(e^{it}) = \int_{\mathbb{D}} \frac{1 - |z|^2}{|1 - ze^{-it}|^2} d\mu(z),$$

is called the balayage (or sweep) of μ. One of the most important results about the balayage states that if μ is a Carleson measure for the Hardy spaces, then S_μ belongs to $BMO(\mathbb{T})$. Recently, an analogue of the balayage in the context of Bergman space, the so-called B-balayage, was introduced by Hasi Wulan, Jun Yang and Kehe Zhu [1]. For a finite positive Borel measure μ on $\mathbb{D}$, the B-balayage of μ is given by

$$G_\mu(z) = \int_{\mathbb{D}} \frac{(1-|a|^2)^2}{|1-a\bar{z}|^4} d\mu(a), \quad z \in \mathbb{D}.$$

In [1] the authors prove that if μ is Carleson measure for the Bergman space, then

$$|G_\mu(z) - G_\mu(w)| \leq \beta(z,w),$$

where β is the hyperbolic metric on $\mathbb{D}$. In the talk we present some extensions of these results on balayage and B-balayage operators.

The talk is based on joint work with Maria Nowak.

[1] Hasi Wulan, Jun Yang and Kehe Zhu: Balayage for the Bergman space *Complex Var. Elliptic Equ.*, 59 (2014), no. 12, 1775 -1782.

Generalized Schur complementation

Zsigmond Tarcsay

Based on a generalized Krein-von Neumann extension theorem we develop a Schur complementation theory in the setting of so-called anti-dual pairs. This allows us to introduce, in a fairly general context, the concept of parallel sum and parallel difference, which play an extremely important role in non-commutative integration theory.

The talk is based on joint work with T. Titkos.

Beyond truncated Toeplitz operators

Dan Timotin

For an inner function $\theta \in H^\infty$, the model space K_θ is defined by the formula $K_\theta = H^2 \ominus \theta H^2$. Truncated Toeplitz operators (TTOs), formally introduced by Sarason more than a decade ago, are compressions to these model spaces of multiplication operators. This area has recently been the focus of extensive research.

In this talk we explore some directions of research that extend the theory of TTOs, namely:

- Considering θ an arbitrary function in the unit ball of H^∞.
- Assuming θ is a matrix-valued inner function.
- As a particular aspect of the previous point, investigating maximal algebras of block Toeplitz matrices.

The talk is based on joint works with H. Bercovici, R. Khan, and M.A. Khan.

Generalized Krein – von Neumann extension

Tamás Titkos

Our long term plan is to develop a unified approach to prove decomposition theorems in different structures. In our anti-dual pair setting, it would be useful to have a tool which is analogous to the so-called Schur complementation. To this aim, I will present a suitable generalization of the classical Krein - von Neumann extension.

The talk is based on joint work with Zs. Tarcsay (Eötvös Loránd University).

Toeplitz operators on Hardy spaces with piecewise continuous symbols

Jani Virtanen

The geometric descriptions of the (essential) spectra of Toeplitz operators with piecewise continuous symbols are among the most beautiful results about Toeplitz operators on Hardy spaces H^p with $1 < p < \infty$, see [1]. In H^1, the essential spectra of Toeplitz operators are known for continuous symbols [3] and symbols in the Douglas algebra $C + H^\infty$ [2]. It is natural to ask whether the theory for piecewise continuous symbols can also be extended to the Hardy space H^1. We answer this question and discuss some other related properties of Toeplitz operators on H^1.

The talk is based on joint work with Santeri Miihkinen.

[1] A. Böttcher and Yu. I. Karlovich: *Carleson curves, Muckenhoupt weights and Toeplitz operators.* Springer, 1997.

[2] M. Papadimitrakis and J. Virtanen: Hankel and Toeplitz transforms on H^1: continuity, compactness and Fredholm properties, *Integr. equ. oper. theory*, 61 (2008), 573–591.

[3] J. Virtanen, Fredholm theory of Toeplitz operators on the Hardy space H^1, *Bull. London Math. Soc.*, 38 (2006), 143–155.

The one-dimensional p-Laplacian with indefinite weight

Bruce Watson

Eigenvalue problems for the one dimensional p-Laplacian on a finite interval,

$$-(|y'(x)|^{p-1}\mathrm{sgn}y'(x))' = (p-1)(\lambda r(x) - q(x))|y(x)|^{p-1}\mathrm{sgn}y(x),$$

for $1 < p < \infty$, will be considered. Here we allow the weight r to be locally integrable and indefinite. Prüfer angle and variational techniques are used. The above considerations leads naturally to the definition and study of the complex p-Laplacian, and the exploration of the Jordan structure of the eigenspaces for this non-linear problem.

This talk is based on joint work with Paul Binding and Patrick Browne.

Canonical commutation relation, traces and affiliated operators

Adam Wegert

The relation $AB - BA = 1$ has its roots in quantum mechanics and is related to the so called Heisenberg uncertainty principle. It is well known that this relation can not hold in any normed algebra. For the algebra of matrices it can be shown easily by applying the trace to both sides of the equality. However one can find *unbounded* operators satisfying this relation. It is natural to ask whether A and B may be chosen to lie in some reasonable algebra. The appropriate notion of such algebra goes back to the work of Murray and von Neumann: this is the so called algebra of operators affiliated with a finite von Neumann algebra. Several years ago it was proven in [1] that it is impossible to realize A, B as operators lying in the algebra of operators affiliated with a finite von Neumann algebra. One can wonder whether it is possible to prove this using the suitable defined trace on such an algebra. We will construct such trace for the case of type I von Neumann algebra and explain what happens in the type II_1 case.

The talk is based on joint work with P. Niemiec.

[1] Z. Liu: On some mathematical aspects of the Heisenberg relation, *Sci. China*, 54 (2011), 2427–2452.

[2] P. Niemiec, A. Wegert: Algebra of operators affiliated with the finite type I von Neumann algebra, *Univ. Iagel. Acta Math.*, 53 (2016), 39–57.

The Crouzeix conjecture and deformations of the numerical range

Michał Wojtylak

Crouzeix observed in [1] that for any operator A in a Hilbert space and any polynomial p

$$\|p(A)\| \leq C \sup_{W(A)} |p|$$

where $W(A)$ is the numerical range of A and the constant $C > 0$ is universal, i.e. does not depend neither on the operator nor on the space. He also proved in the same paper that $2 \leq C \leq 11.08$ and conjectured that $C = 2$. We will review recent developments on proving the conjecture ($C \leq 1 + \sqrt{2}$ in [2]) and show some deformations of the numerical range that lead to new constants.

The talk is based on joint work with P. Pagacz and P. Pietrzycki.

[1] Crouzeix, Michel: Numerical range and functional calculus in Hilbert space, *Journal of Functional Analysis*, 244.2 (2007), 668-690.

[2] Crouzeix, Michel, and César Palencia: The Numerical Range is a $(1 + \sqrt{2})$-Spectral Set, *SIAM Journal on Matrix Analysis and Applications*, 38.2 (2017), 649-655.

Spectral resolutions for non-self-adjoint block Jacobi matrices

Ewelina Zalot

The purpose is to present spectral resolutions for a class of non-self-adjoint Toeplitz type operators. Applications to periodic Jacobi matrices are also considered.

On the spectrum of the quantum Rabi model

Lech Zielinski

The quantum Rabi model is the simplest physical model of the interaction between radiation and matter which is a central problem in quantum optics ([1]). The Hamiltonian of this model is a self-adjoint operator with discrete spectrum. We present its definition, basic properties and a rigorous statement concerning the procedure which is called GRWA (Generalized Rotating-Wave Approximation) in the physical literature following [2]. We show how our result allows one to solve the corresponding inverse problem, i.e. to reconstruct all parameters of the model from the spectrum.

The talk is based on joint work with A. Boutet de Monvel.

[1] D. Braak, Q.-H. Chen, M. T. Batchelor, and E. Solano: Semi-classical and quantum Rabi models: in celebration of 80 years, *J. Phys. A: Math. Theor.*, 49 (2016), no. 30, 30030

[2] E. K. Irish: Generalized rotating-wave approximation for arbitrarily large coupling, *Phys. Rev. Lett.* 99 (2007), 173601

Spectral Analysis of Block Jacobi Matrices with Singular Entries

Marcin J. Zygmunt

Three-diagonal block matrices (BM) are strictly connected with matrix-valued orthogonal polynomials (MOP). Methods arising from the theory of those MOP gives answers to questions about spectrum of corresponding BM.

The case of singular coefficients in matrix-valued three-term recurrence formula such MOP satisfy will be treated. The results will be applied to give another approach to the analysis of classic orthogonal polynomials satisfying three-term recurrence formula with periodic coefficients.

[1] M. J. Zygmunt: Matrix orthogonal polynomials and continued fractions, *Linear Alg. Appl.*, 340 1-3 (2002), 155–168.

[2] M. J. Zygmunt: Jacobi Block Matrices with Constant Matrix Terms, *Oper. Theory Adv. Appl.*, 154 (2004), 233–238.

[3] M. J. Zygmunt: Matrix orthogonal polynomials with respect to a non-symmetric matrix of measures, *Opuscula Math.*, 36 3 (2016), 409–423.